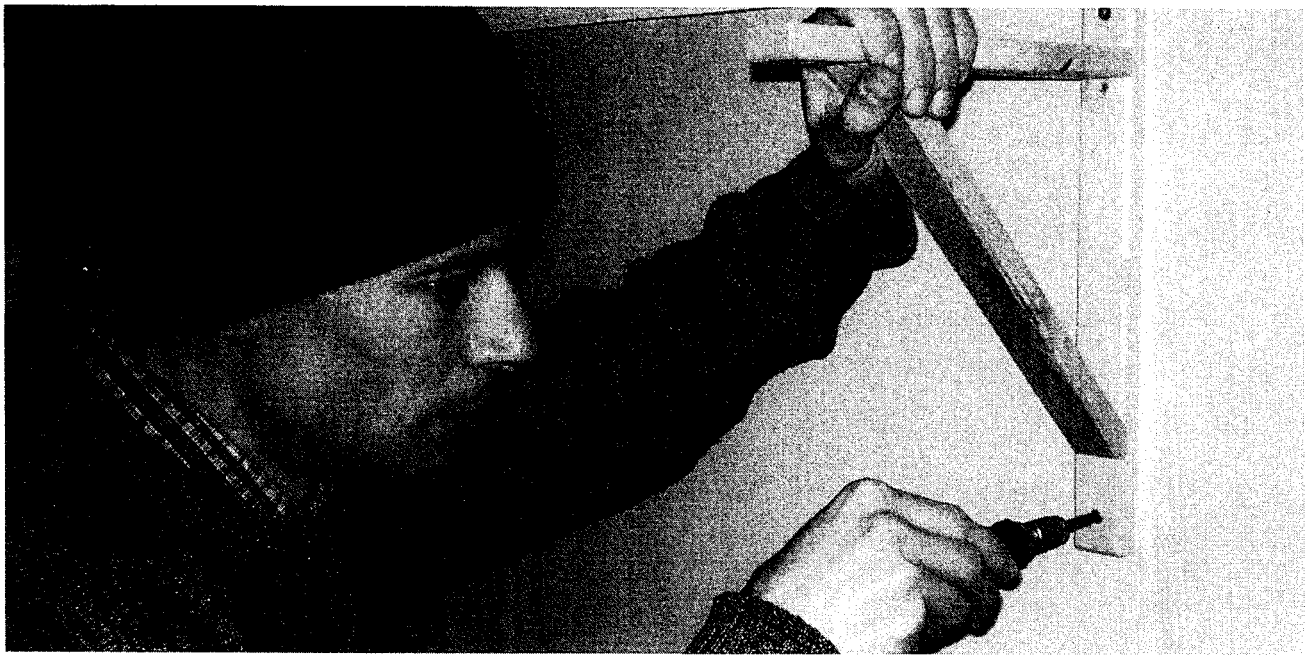
Chapter 7

Trigonometry of Right Triangles



This man is installing an angle brace. By incorporating right triangles, angle braces provide support and strength to bookshelves, building roofs, and beyond.

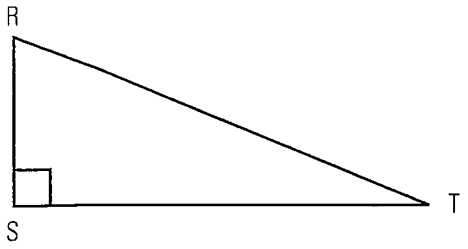
7.1 The Pythagorean Theorem

REVIEW: WORKING WITH TRIANGLES

Each vertex of a triangle is labelled with an upper case letter, and each side is labelled either with the lower case letter corresponding to the opposite vertex or with the upper case letters of the vertices it connects.

Example 1

Consider ΔRST.

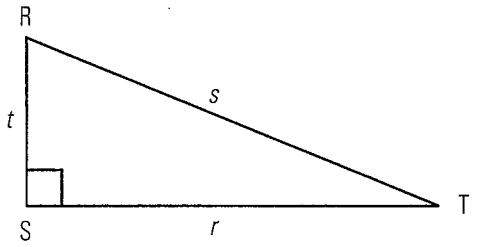


a) Label the sides with the appropriate lower case letter.

b) Name the sides using the upper case letters of the vertices they connect.

SOLUTION

a) Each side is labelled with the lower case letter corresponding to the opposite vertex.



b) The sides can also be named according to the upper case letters of the vertices they connect.

Side r can be called side ST.

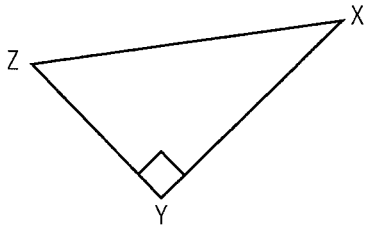
Side s can be called side TR.

Side t can be called side RS.

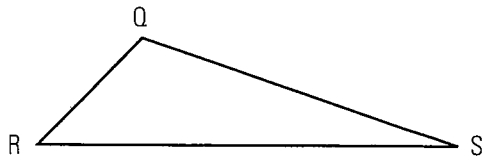
BUILD YOUR SKILLS

1. Label each side of the triangles below using a single lower case letter corresponding to the opposite vertex.

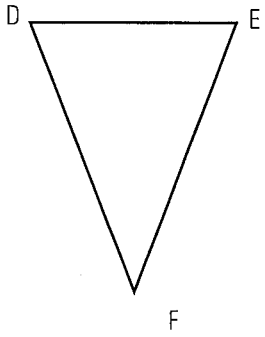
a)



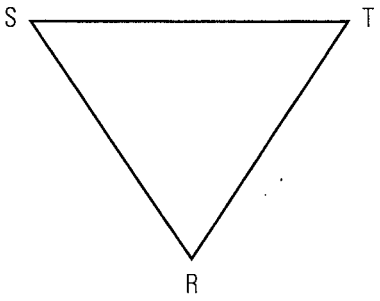
b)



c)

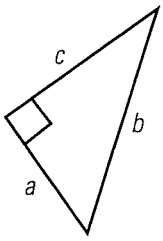


d)

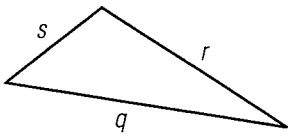


2. Label each vertex of the triangles below using a single upper case letter corresponding to the opposite side.

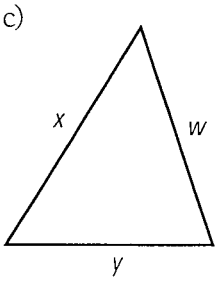
a)



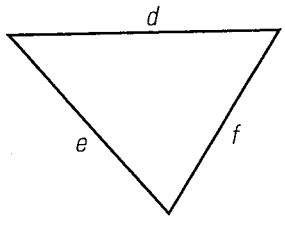
b)



c)



d)



NEW SKILLS: WORKING WITH THE PYTHAGOREAN THEOREM

A right triangle is a triangle with one right angle. The side opposite the right angle is the longest side and is called the **hypotenuse**. The other two sides are called legs (or, in some cases, arms).

hypotenuse: the long side of a right triangle, opposite the 90° angle

The **Pythagorean theorem** states the relationship among the sides of a right triangle. Given a right triangle ABC with right angle C, the Pythagorean theorem states the following.

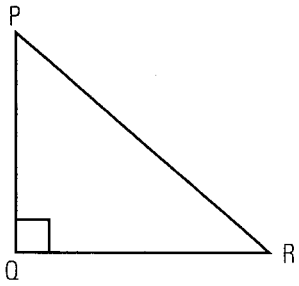
Pythagorean theorem: in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse

$a^2 + b^2 = c^2$

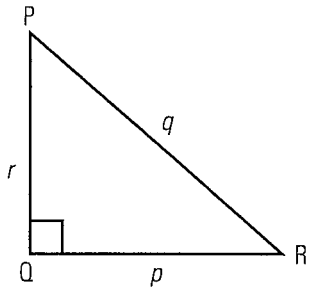
For more details, see page 272 of *MathWorks 10*.

Example 2

Label the sides of the triangles and state the Pythagorean theorem as it applies to them.



SOLUTION

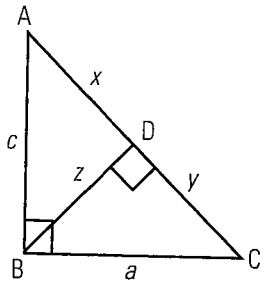


Since q is the hypotenuse, the Pythagorean theorem is written as follows.

$p^2 + r^2 = q^2$

BUILD YOUR SKILLS

3. Given the following diagram, use the lettering provided to state three Pythagorean relations that apply.



4. A ladder, ℓ, is placed against the side of a house, h. The foot of the ladder is a distance d from the base of the house. Draw a diagram and express the relationship that exists between ℓ, h, and d.

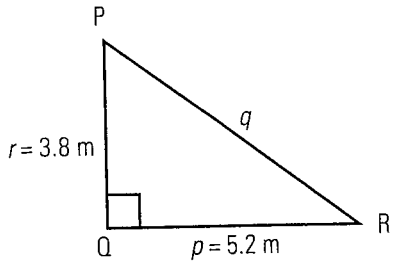
5. Rearrange the Pythagorean theorem to solve first for x and then for y.

$x^2 + y^2 = z^2$

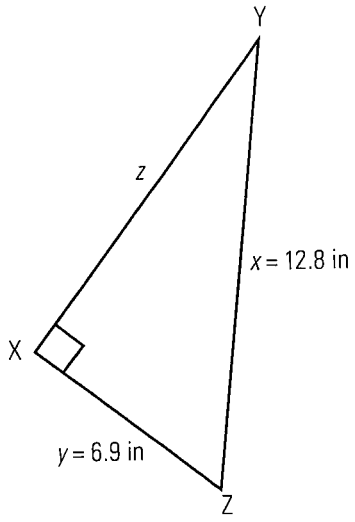
Example 3

Use the Pythagorean theorem to find the lengths of the missing sides of the triangles to the nearest tenth of a unit.

a)



b)



SOLUTION

a) Write the Pythagorean theorem using the labels on the given triangle.

$$p^2 + r^2 = q^2$$

$5.2^2 + 3.8^2 = q^2$ Substitute the known values.

$$27.04 + 14.44 = q^2$$

$$41.48 = q^2$$

$\sqrt{41.48} = q$ Take the square root of both sides.

$$6.44 \approx q$$

Side q is approximately 6.4 m.

b) Write the Pythagorean theorem using the labels on the given triangle.

$$y^2 + z^2 = x^2$$

$6.9^2 + z^2 = 12.8^2$ Substitute the known values.

$$47.61 + z^2 = 163.84$$

$z^2 = 163.84 - 47.61$ Subtract 47.61 from both sides to isolate z.

$$z^2 = 116.23$$

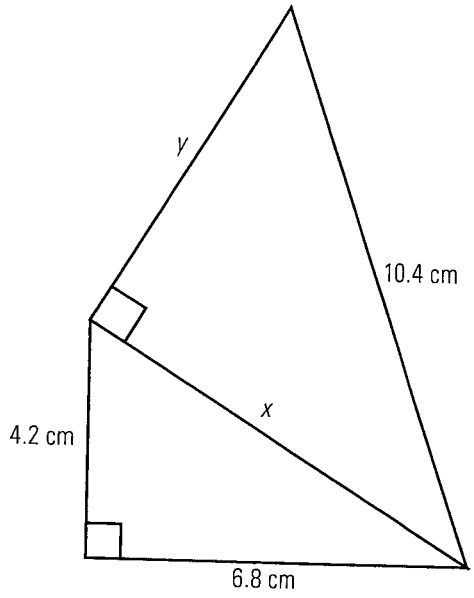
$z = \sqrt{116.23}$ Take the square root of both sides.

$$z \approx 10.78$$

Side z is approximately 10.8 inches.

BUILD YOUR SKILLS

6. Calculate the values of x and y.



7. A 40-foot ladder reaches 38 feet up the side of a house. How far from the base of the house is the foot of the ladder?

8. A field is 120 m by 180 m. How much shorter is your route if you walk diagonally across the field rather than walking around the edge to the opposite corner?

PRACTISE YOUR NEW SKILLS

1. A stairway rises 6 feet 4 inches over a horizontal distance of 8 feet 6 inches. What is the diagonal length of the stairway?

2. A 28-metre long guy wire is attached to a point 24 m up the side of a tower. How far from the base of the tower is the guy wire attached?

3. The construction plans for a ramp show that it rises 3.5 metres over a horizontal distance of 10.5 metres. How long will the ramp surface be?

4. The advertised size of a TV screen is the distance between opposite corners. Sally bought a 52-inch TV. If the height of the TV is 32 inches, how wide is it?

5. A boat sailed due north at a rate of 12 km/h for 3 hours, then due east at a rate of 18 km/h for 2 hours. How far was it from its starting point, measuring the shortest distance?

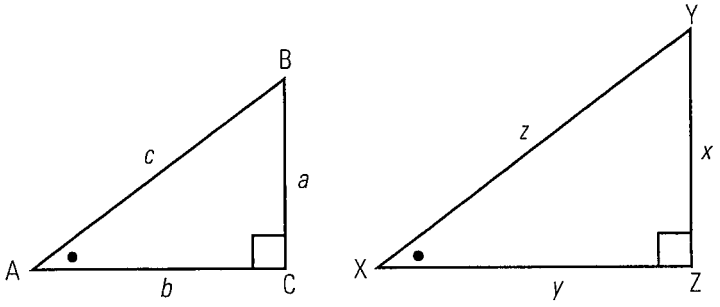
7.2 The Sine Ratio

NEW SKILLS: WORKING WITH THE SINE RATIO TO SOLVE TRIANGLES

In chapter 6, you worked with similar triangles to discover that, in triangles with congruent angles, the ratio between the corresponding sides of the similar triangles is the same.

The following diagram shows similar triangles. $\Delta ABC \sim \Delta XYZ$.

Angles marked with the same symbol are equal.



The ratios between corresponding sides are equal, so we know that the following is true.

$$\frac{a}{x} = \frac{c}{z}$$

This proportion can be rearranged so that each side of the equation represents a ratio of sides from the same triangle.

$$\cancel{x} \times z \times \frac{a}{\cancel{x}} = \frac{c}{\cancel{z}} \times \cancel{z} \times x$$

Multiply both sides by the product of the denominators and simplify.

$$za = cx$$

$$\frac{\cancel{z}a}{c\cancel{z}} = \frac{\cancel{z}x}{\cancel{z}z}$$

Divide both sides by the same number and simplify.

$$\frac{a}{c} = \frac{x}{z}$$

When triangles are similar, the ratio of the length of the side opposite a given angle to the length of the hypotenuse is always the same. This ratio is referred to as the **sine ratio**.

sine ratio: in a right triangle, the ratio of the length of the side opposite a given angle to the length of the hypotenuse (abbreviated as sin)

Given any right triangle with acute angle A, the sine ratio can be written as follows.

$$\sin \angle A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

The ratio is abbreviated as follows.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

Use your scientific calculator to calculate the values of the sines of angles.

For more details, see page 283 of *MathWorks 10.*

Example 1

Use your calculator to determine the following sine ratios. Round to four decimal places.

a) sin 15°

b) sin 30°

c) sin 60°

d) sin 80°

What do you notice about these values?

SOLUTION

a) sin 15° = 0.2588

b) sin 30° = 0.5000

c) sin 60° = 0.8660

d) sin 80° = 0.9848

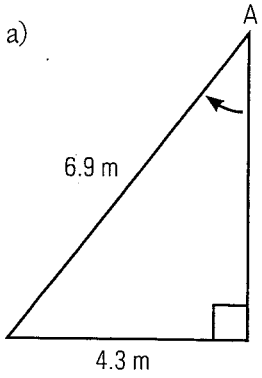
The sine ratio determines that if you have a right triangle with an acute angle given, regardless of the size of the triangle, the ratio of the side opposite that angle to the hypotenuse will always be the same.

The value of the sine ratio increases as the angle gets bigger.

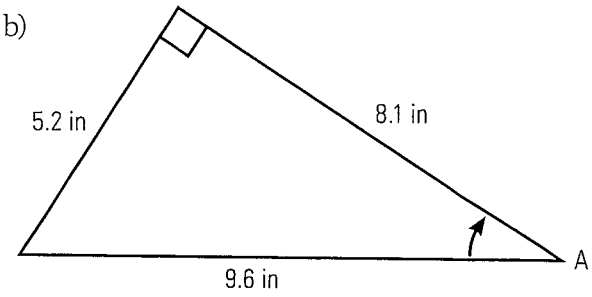
BUILD YOUR SKILLS

1. Calculate the value of sin A to two decimal places.

a)



b)



2. Use your calculator to determine the value of each of the following sine ratios to four decimal places.

a) sin 10°

b) sin 48°

c) sin 62°

d) sin 77°

3. Use your calculator to determine the value of sin 90°. Suggest a reason why this is so.

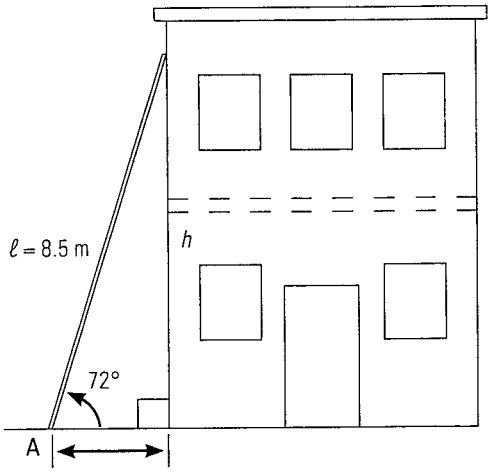
Example 2

The sine ratio can be used to help you find missing parts of a right triangle.

A ladder 8.5 metres long makes an angle of 72° with the ground. How far up the side of a building will it reach?

SOLUTION

Sketch a diagram.



The height, h, is opposite the 72° angle, and the ladder, ℓ, forms the hypotenuse of the triangle. A right triangle is formed, with h as the side opposite the 72° angle, and ℓ as the hypotenuse.

Use the sine ratio to calculate h.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 72° = \frac{h}{8.5}$$ Substitute the known values.

$$8.5 \times \sin 72° = \frac{h}{8.5} \times 8.5$$ Multiply both sides by 8.5.

$$8.5 \times \sin 72° = h$$

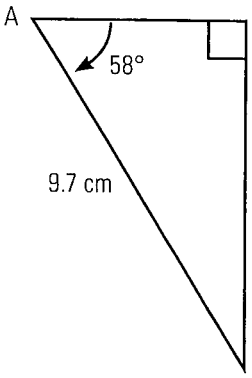
$$8.1 \approx h$$

The ladder reaches approximately 8.1 metres up the side of the building.

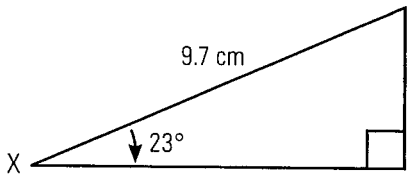
BUILD YOUR SKILLS

4. Calculate the length of the side opposite the indicated angle in the following diagrams.

a)



b)



5. A rafter makes an angle of 28° with the horizontal. If the rafter is 15 feet long, what is the height at the rafter's peak?

6. How high is a weather balloon tied to the ground if it is attached to a 15-metre string and the angle between the string and the ground is 35°?

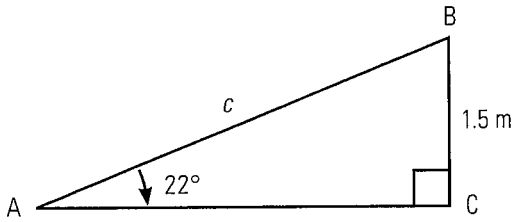
Example 3

Brad is building a ramp. The ramp must form an angle of 22° with the level ground and reach a point that is 1.5 metres above the ground. How long will the ramp be?

Wait until you have isolated the unknown variable before doing the calculation. This will minimize errors due to rounding.

SOLUTION

Sketch a diagram.



Let c represent the length of the ramp. On the diagram, 1.5 metres is opposite the 22° angle.

Use the sine ratio to solve for c.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 22° = \frac{1.5}{c}$$ Substitute the known values.

$$c \times \sin 22° = \frac{1.5}{c} \times c$$ Multiply both sides by c.

$$c \times \sin 22° = 1.5$$ Simplify.

$$\frac{c \times \sin 22°}{\sin 22°} = \frac{1.5}{\sin 22°}$$ Divide both sides by sin 22° to isolate c.

$$c = \frac{1.5}{\sin 22°}$$

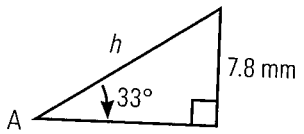
$$c \approx 4.004$$

The ramp is approximately 4 metres long.

BUILD YOUR SKILLS

7. Find the length of the hypotenuse in the following diagrams.

a)



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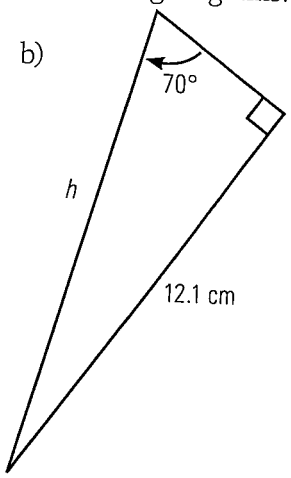


8. How long is a guy wire that is attached 4.2 metres up a pole if it makes an angle of 52° with the ground?

9. A boat is carried with the current at an angle of 43° to the shore. If the river is approximately 15 metres wide, how far does the boat travel before reaching the opposite shore?

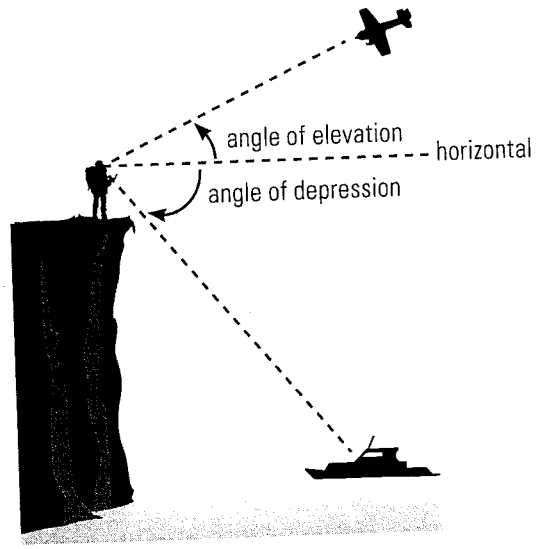
NEW SKILLS: WORKING WITH ANGLE OF ELEVATION AND DEPRESSION

When you look up at an airplane flying overhead, the angle between the horizontal and your line of sight is called an **angle of elevation.** When you look down from a cliff to a boat passing by, the angle between the horizontal and your line of sight is called an **angle of depression**.

angle of elevation: the angle formed between the horizontal and the line of sight while looking upward; sometimes referred to as the angle of inclination

angle of depression: the angle formed between the horizontal and the line of sight when looking downward

For more details, see pages 288–289 of *MathWorks 10*.

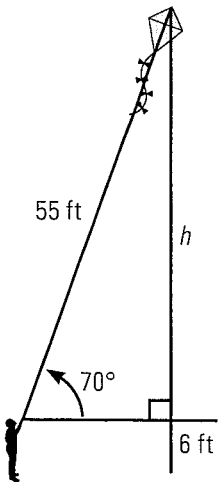


Example 4

The angle of elevation of Sandra's kite string is 70°. If she has let out 55 feet of string, and is holding the string 6 feet above the ground, how high is the kite?

SOLUTION

Sketch and label a diagram.



Use the sine ratio to solve for the height of the kite.

$$\sin H = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 70° = \frac{h}{55}$$ Substitute the known values.

$$55 \times \sin 70° = \frac{h}{55} \times 55$$ Multiply both sides by 55.

$$55 \times \sin 70° = h$$

$$51.7 \approx h$$

The kite is approximately 52 feet above where Sandra is holding it. Add 6 feet for the distance between the ground and the start of the string. The kite is about 58 feet above the ground.

BUILD YOUR SKILLS

10. George is in a hot air balloon that is 125 metres high. The angle of elevation from a house below, to the balloon, is 18°. How far is George from the house?

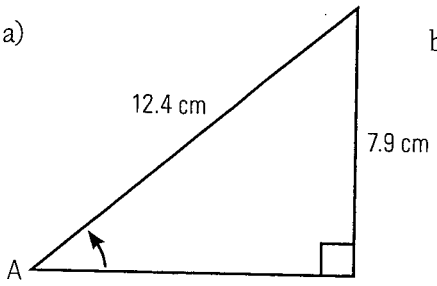
11. The angle of elevation of a road is 4.5°. What is the length of the section of road if it rises 16 metres?

12. The angle of elevation of a slide that is 3.6 metres long is 32°. How high above the ground is the top of the slide?

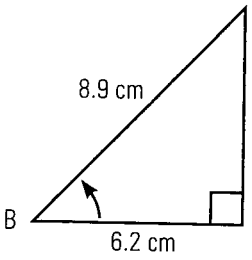
PRACTISE YOUR NEW SKILLS

1. Calculate the sine of the indicated angle.

a)

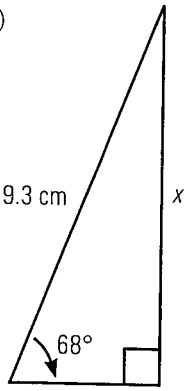


b)



2. Calculate the length of the indicated side.

a)



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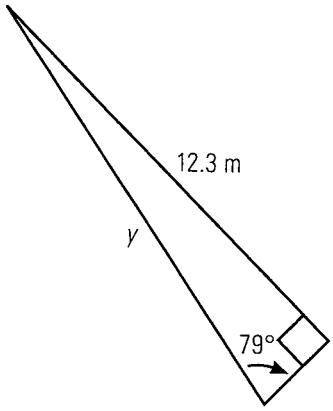


3. A ramp with a length of 21.2 metres has an angle of elevation of 15°. How high up does it reach?

4. The angle of elevation from the bottom of a waterslide to the platform above is 20°. If the waterslide is 25 metres long, how high is the platform?

5. A man walks at an angle of 68° north of east for 45 metres. How far north of his starting point is he?

7.3 The Cosine Ratio

NEW SKILLS: WORKING WITH THE COSINE RATIO TO SOLVE TRIANGLES

cosine ratio: in a right triangle, the ratio of the length of the side adjacent a given angle to the length of the hypotenuse (abbreviated as cos)

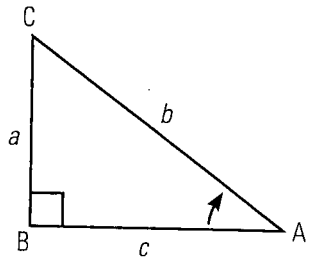
Another important trigonometric ratio of right triangles is the ratio of the side adjacent to the given acute angle to the hypotenuse. This is called the **cosine ratio.**

For a given angle A, the cosine ratio can be stated as follows.

$$\text{cosine } \angle A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$$

This ratio can be abbreviated as follows.

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$



For triangle ABC, the cosine of $\angle A$ can be stated as the following.

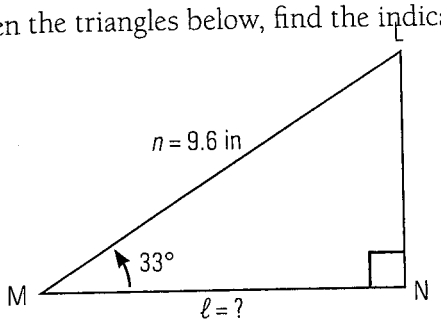
$$\cos A = \frac{c}{b}$$

For more details, see page 293 of *MathWorks 10.*

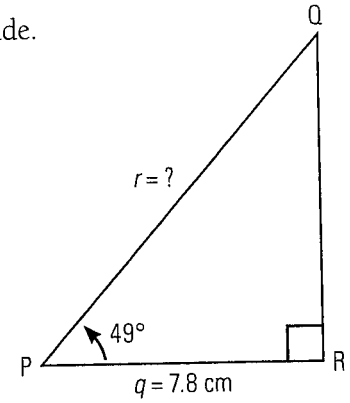
Example 1

Given the triangles below, find the indicated side.

a)



b)



SOLUTION

a) Use the cosine ratio to solve for ℓ.

$$\cos M = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 33° = \frac{\ell}{9.6}$$ Substitute the known values.

$$9.6 \times \cos 33° = \frac{\ell}{9.6} \times 9.6$$ Multiply both sides by 9.6 to isolate ℓ.

$$9.6 \cos 33° = \ell$$ Simplify.

$$8.05 \approx \ell$$

The missing side is approximately 8.1 inches long.

The notation $9.6 \cos 33°$, without the multiplication symbol, can be used to mean $9.6 \times \cos 33°$.

ALTERNATIVE SOLUTION

a) Since you know that the sum of the angles of a triangle is 180°, you can calculate angle L and then use the sine ratio to solve for ℓ.

$$180° - 90° - 33° = 57°$$

$$\sin L = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 57° = \frac{\ell}{9.6}$$ Substitute the known values.

$$9.6 \sin 57° = \frac{\ell}{9.6} \times 9.6$$ Multiply both sides by 9.6 to isolate ℓ.

$$9.6 \sin 57° = \ell$$

$$8.05 \approx \ell$$

Side ℓ is approximately 8.1 inches long.

b) Use the cosine ratio to solve for r.

$$\cos P = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 49° = \frac{7.8}{r}$$ Substitute the known values.

$$r \cos 49° = \frac{7.8}{r} \times r$$ Multiply both sides by r.

$$r \cos 49° = 7.8$$ Simplify.

$$\frac{r \cos 49°}{\cos 49°} = \frac{7.8}{\cos 49°}$$ Divide both sides by cos 49° to isolate r.

$$r \approx 11.89$$

Side r is approximately 11.9 cm long.

BUILD YOUR SKILLS

1. Use your calculator to find the following pairs of ratios to four decimal places.

a) $\cos 23° =$

$\sin 67° =$

b) $\cos 83° =$

$\sin 7° =$

c) $\cos 45° =$

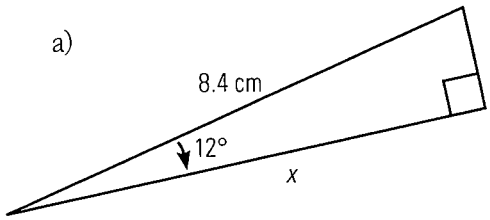
$\sin 45° =$

d) $\cos 37° =$

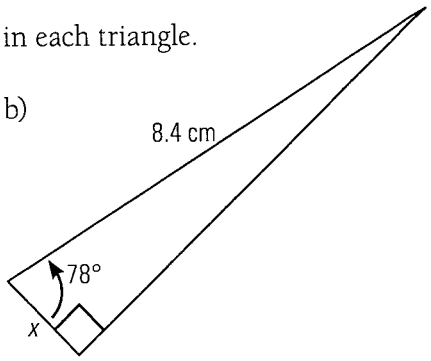
$\sin 53° =$

2. Find the measure of the indicated side in each triangle.

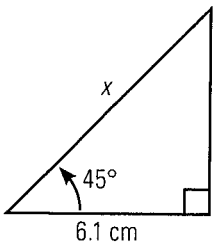
a)



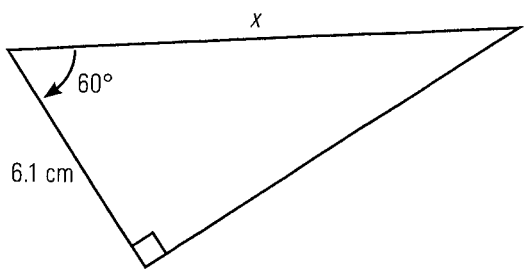
b)



c)



d)

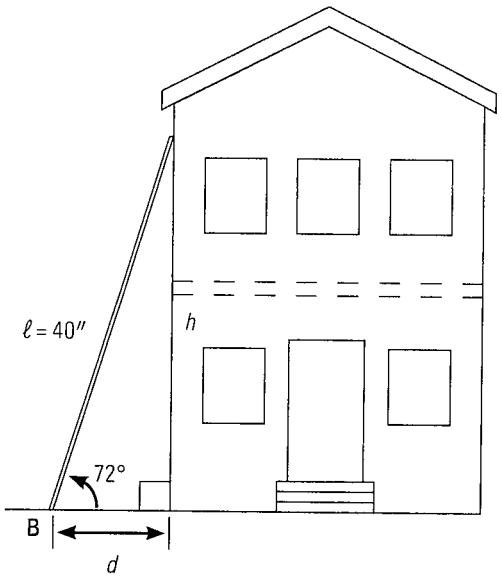


Example 2

How far from the base of a house is a 40-foot ladder if the angle of elevation is 72°?

SOLUTION

Sketch a diagram.



Use the cosine ratio to solve for d.

$$\cos B = \frac{\text{adj}}{\text{hyp}}$$

$$\cos B = \frac{d}{\ell}$$

$$\cos 72° = \frac{d}{40}$$ Substitute the known values.

$$40 \cos 72° = \frac{d}{40} \times 40$$ Multiply both sides by 40.

$$40 \cos 72° = d$$ Simplify.

$$12.36 \approx d$$

The ladder rests about 12.4 feet from the house.

BUILD YOUR SKILLS

3. How far from the base of a flagpole must a guy wire be fixed if the wire is 12 metres long and it makes an angle of 63° with the ground?

4. Reba walks 25 yards across the diagonal of a rectangular field. If the angle between the width and the diagonal is 67°, how wide is the field?

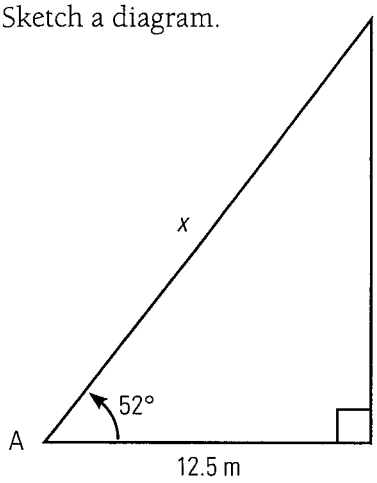
5. A square pyramid has a slant height of 9 metres. The slant height makes an angle of 70° with the ground. What is the length of a side of the pyramid?

Example 3

The angle of a cable from a point 12.5 metres from its base is 52°. How long is the cable?

SOLUTION

Sketch a diagram.



Use the cosine ratio to solve for x, the length of the cable.

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 52° = \frac{12.5}{x}$$ Substitute the known values.

$$x \cos 52° = \frac{12.5}{x} \times x$$ Multiply both sides by x.

$$x \cos 52° = 12.5$$ Simplify.

$$\frac{x \cos 52°}{\cos 52°} = \frac{12.5}{\cos 52°}$$ Divide both sides by cos 52° to isolate x.

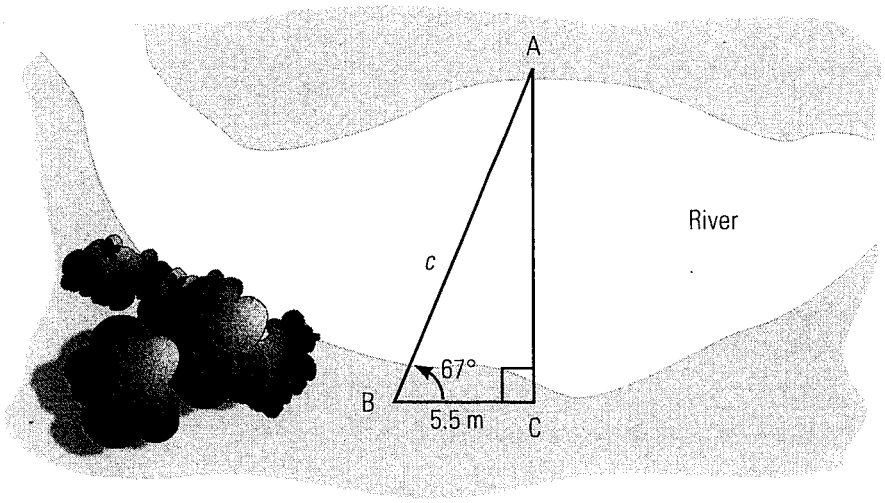
$$x = \frac{12.5}{\cos 52°}$$

$$x \approx 20.30$$

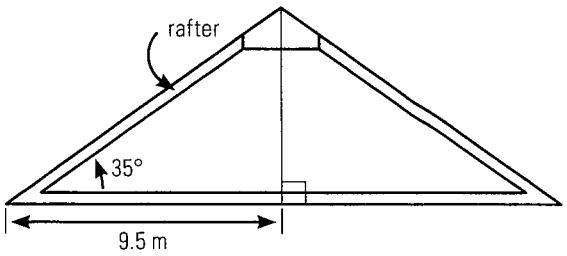
The cable is approximately 20.3 metres long.

BUILD YOUR SKILLS

6. Arul needs to string a bridge line across the river from A to B. What must the length of the bridge line be, given his measurements?



7. What is the length of a rafter that makes an angle of 35° with the floor of an attic whose centre is 9.5 metres from the edge?

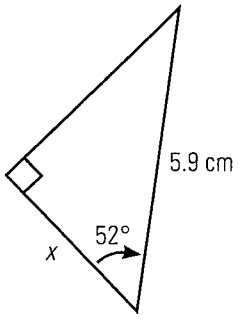


8. An airplane starts descending at an angle of depression of 5°. If the horizontal distance to its destination is 500 kilometres, what is the actual distance the airplane will travel before it lands?

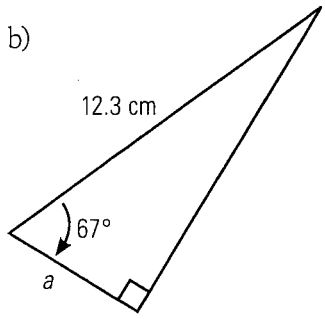
PRACTISE YOUR NEW SKILLS

1. Find the lengths of the indicated sides.

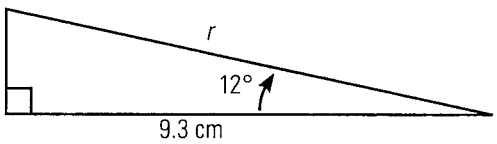
a)



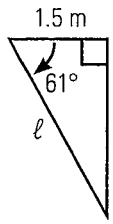
b)



c)



d)



2. A screw conveyor is sometimes used to move grains and other materials up an incline. How far from the base of a barn must a 20-metre screw conveyor be placed if the angle of elevation to the loft is to be 30°?

3. What is the slant height of a cone if the diameter is 20 centimetres and the angle made with it is 65°?

4. A hot air balloon travels 1.2 kilometres horizontally from its take-off point. The angle of elevation from the take-off point to the balloon is 15°. How far did the balloon travel?

5. What horizontal distance has a car travelled if the incline of the road averages 3.2° and the car's odometer reads 8.5 kilometres?

6. The horizontal distance between two clothesline poles is 3.4 metres. When wet clothes are hung in the middle of the line, it sags at an angle of depression of 6°. How long is the clothesline?

7.4 The Tangent Ratio

NEW SKILLS: WORKING WITH THE TANGENT RATIO TO SOLVE TRIANGLES

tangent ratio: in a right triangle, the ratio of the length of the side opposite a given angle to the length of the side adjacent to the angle (abbreviated as tan)

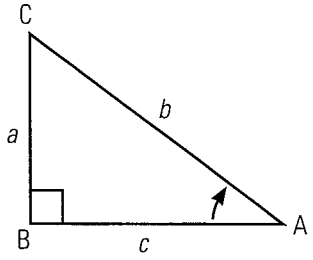
You have studied two trigonometric ratios, the sine ratio and the cosine ratio. The third trigonometric ratio is the **tangent ratio.**

The tangent ratio is defined as the ratio of the side opposite an acute angle of a right triangle to the side adjacent the angle. For angle A, the ratio can be stated as follows.

$$\text{tangent } \angle A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$

This can be abbreviated as the following ratio.

$$\tan A = \frac{\text{opp}}{\text{adj}}$$



For triangle ABC, the tangent of angle A can be stated as follows.

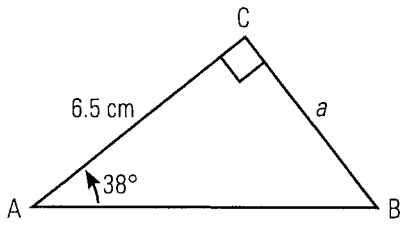
$$\tan A = \frac{a}{c}$$

For more details, see page 301 of *MathWorks 10.*

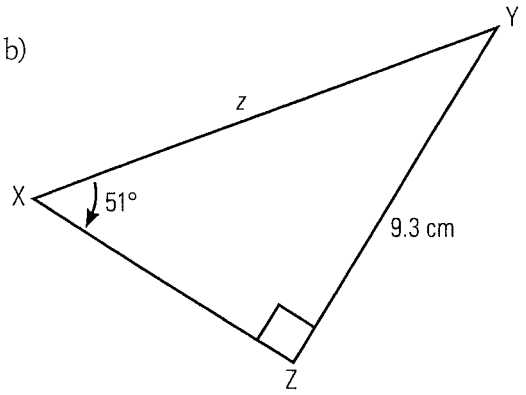
Example 1

Find the indicated side of each triangle.

a)



b)



SOLUTION

a) Use the tangent ratio to solve for a.

$$\tan A = \frac{\text{opp}}{\text{adj}}$$

$$\tan 38° = \frac{a}{6.5}$$ Substitute the known values.

$$6.5 \tan 38° = \frac{a}{6.5} \times 6.5$$ Multiply both sides by tan 6.5 to isolate a.

$$6.5 \tan 38° = a$$

$$5.08 \approx a$$

Side a is approximately 5.1 centimetres long.

b) Use the tangent ratio to solve for z.

$$\tan X = \frac{\text{opp}}{\text{adj}}$$

$$\tan 51° = \frac{9.3}{z}$$ Substitute the known values.

$$z \tan 51° = 9.3$$ Multiply both sides by z.

$$\frac{z \tan 51°}{\tan 51°} = \frac{9.3}{\tan 51°}$$ Divide both sides by tan 51° to isolate z.

$$z = \frac{9.3}{\tan 51°}$$

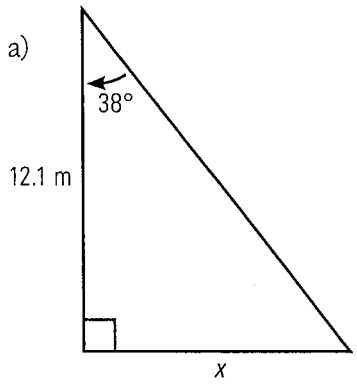
$$z \approx 7.53$$

Side z is approximately 7.5 centimetres long.

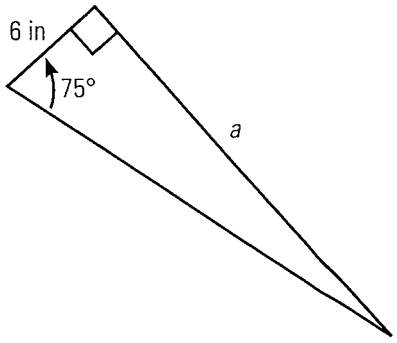
BUILD YOUR SKILLS

1. Find the length of the indicated sides of the triangles.

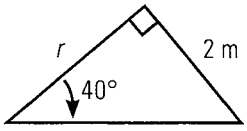
a)



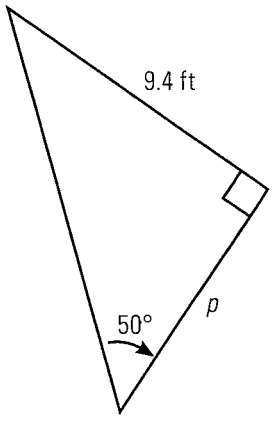
b)



c)

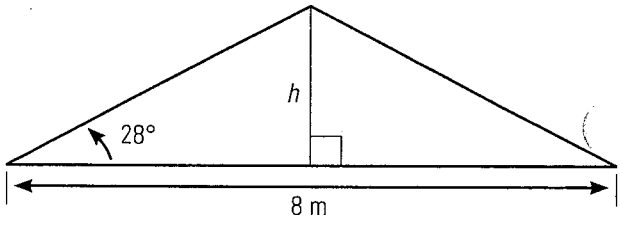


d)



2. The angle of depression to a boat from the top of a 150-metre cliff is 20°. How far is the boat from the base of the cliff?

3. When sand is piled onto a flat surface, it forms a cone. If the pile is 8 m wide, and the angle between the ground and the slope of the pile is 28°, what is the height of the pile?



PRACTISE YOUR NEW SKILLS

1. A 1.7-metre tall man stands 12 m from the base of a tree. He views the top of the tree at an angle of elevation of 58°. How tall is the tree?

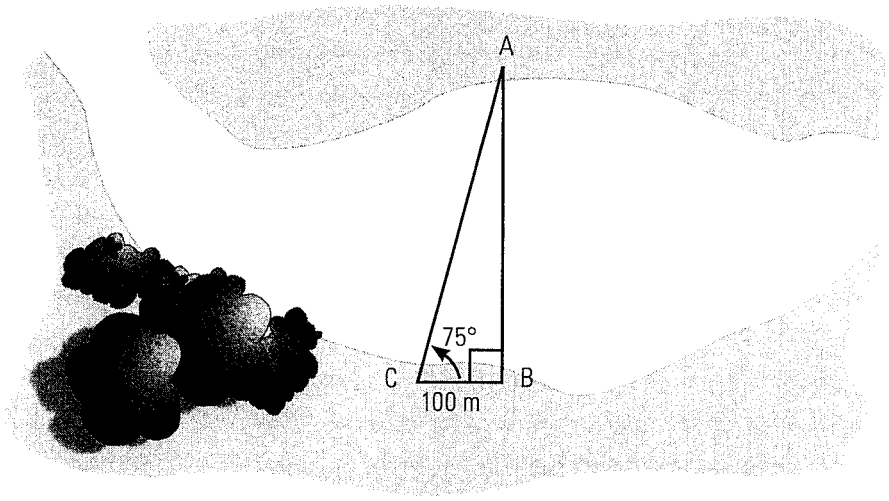
2. Two buildings are 18.5 metres apart. The angle of elevation from the top of one building to the top of the other is 18°. If the taller building is 15 metres tall, how tall is the shorter building?

3. How far from the base of the house is the foot of a ladder if the angle of elevation is 70° and it reaches 15 feet up the side of the house?

4. About how tall is a tower if the angle of depression from its top to a point 75 metres from the base is 62°?

5. A rafter's angle of elevation with the horizontal is 25°. How far from the corner could a 6-foot man stand up straight?

6. Determine the distance, AB, across the river, given the following measurements.



7.5 Finding Angles and Solving Right Triangles

NEW SKILLS: WORKING WITH INVERSE TRIGONOMETRIC RATIOS

The trigonometric ratios discussed in this chapter are unaffected by the size of the triangle, provided that the acute angle remains the same.

If you know the trigonometric ratio, you can calculate the size of the angle. This requires an "inverse" operation. You can use your calculator to find the opposite of the usual ratio calculation. You can think of the inverse in terms of subtraction and addition: subtraction is the inverse, or opposite, of addition because it "undoes" the operation.

For more details, see page 307 of *MathWorks 10.*

Example 1

Calculate each angle to the nearest degree.

a) $\sin A = 0.2546$

b) $\cos B = 0.1598$

c) $\tan C = 3.2785$

SOLUTION

Use the inverse function on your calculator.

a) $\sin A = 0.2546$

$A = \sin^{-1}(0.2546)$

$A \approx 14.7$

$\angle A$ is approximately 15°.

b) $\cos B = 0.1598$

$B = \cos^{-1}(0.1598)$

$B \approx 80.8$

$\angle B$ is approximately 81°.

c) $\tan C = 3.2785$

$C = \tan^{-1}(3.2785)$

$C \approx 73.0$

$\angle C$ is approximately 73°.

BUILD YOUR SKILLS

1. Calculate the angle to the nearest degree.

   a) $\sin D = 0.5491$

   b) $\cos F = 0.8964$

   c) $\tan G = 2.3548$

   d) $\sin H = 0.9998$

2. In right triangle ΔXYZ, the ratio of the side opposite $\angle X$ to the hypotenuse is $\frac{7}{8}$. What is the approximate size of $\angle X$?

When solving this problem on your calculator, put brackets around $\frac{7}{8}$.

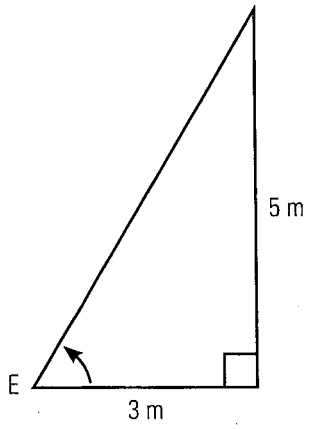
3. What is the approximate size of an angle in a right triangle if the ratio of the side opposite the angle to the side adjacent to the angle is $\frac{15}{8}$?

Example 2

Determine the angle of elevation to the top of a 5-metre tree at a point 3 metres from the base of the tree.

SOLUTION

Sketch a diagram.



When solving this problem on your calculator, put brackets around $\frac{5}{3}$.

You are given the height (h, 5 metres) and the length (ℓ, 3 metres) of the triangle, and you need to solve for the angle of elevation. Use the tangent ratio.

$$\tan E = \frac{\text{opp}}{\text{adj}}$$

$$\tan E = \frac{5}{3}$$ Substitute the known values.

$$E = \tan^{-1}\left(\frac{5}{3}\right)$$ Use the inverse function to solve for E.

$$E \approx 59.0362$$

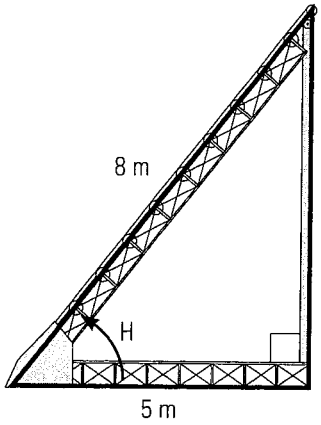
The angle of elevation is approximately 59°.

BUILD YOUR SKILLS

4. What is the angle of depression from the top of a 65-metre cliff to an object 48 metres from its base?

5. At what angle to the ground must you place a support if it is 6.8 metres long and must reach 4.2 metres up the side of a tower?

6. At what angle to the ground is an 8-metre long conveyor belt if it is fastened 5 metres from the base of a loading ramp?

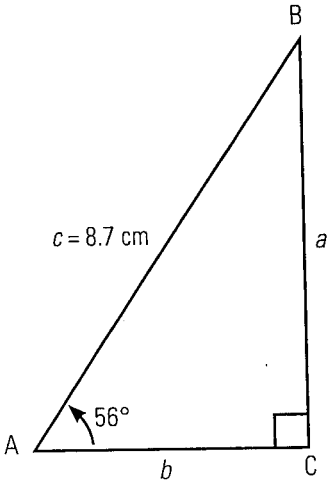


NEW SKILLS: WORKING WITH RATIOS TO SOLVE TRIANGLES

Solving a triangle means finding the values of all the unknown sides and angles. In a right triangle, you already know that one angle is 90°, so there are only five other parts to consider: the three sides, and the two other angles. If you are given any two sides, or any one side and one angle, you can use trigonometry to find the other values.

Example 3

Solve the right triangle. Give lengths to the nearest tenth.



SOLUTION

You are given two of the three angles, so you can solve for the third angle.

$$\angle B = 180° - 90° - 56°$$

$$\angle B = 34°$$

To solve for side a, you can use the sine ratio. Use $\angle A$, and the length of the hypotenuse, c.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin A = \frac{a}{c}$$

$$\sin 56° = \frac{a}{8.7}$$ Substitute the known values.

$$8.7 \sin 56° = \frac{a}{8.7} \times 8.7$$ Multiply both sides by 8.7 to isolate a.

$$7.2126 \approx a$$

Side a is approximately 7.2 cm long.

To solve for b, use the cosine ratio.

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

$$\cos A = \frac{b}{c}$$

$$\cos 56° = \frac{b}{8.7}$$ Substitute the known values.

$$8.7 \cos 56° = \frac{b}{8.7} \times 8.7$$ Multiply both sides by 8.7 to isolate b.

$$8.7 \cos 56° = b$$ Simplify.

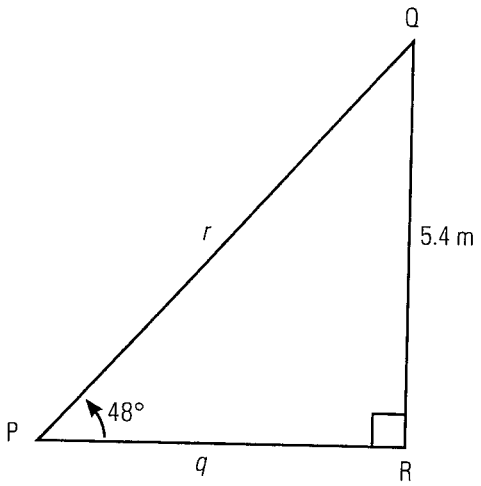
$$4.8650 \approx b$$

Side b is approximately 4.9 cm long.

You could have used the Pythagorean theorem to find side b, but this would have been less accurate because you would have used an approximation for side a. It is always better to use the numbers given, if possible, rather than one you calculated.

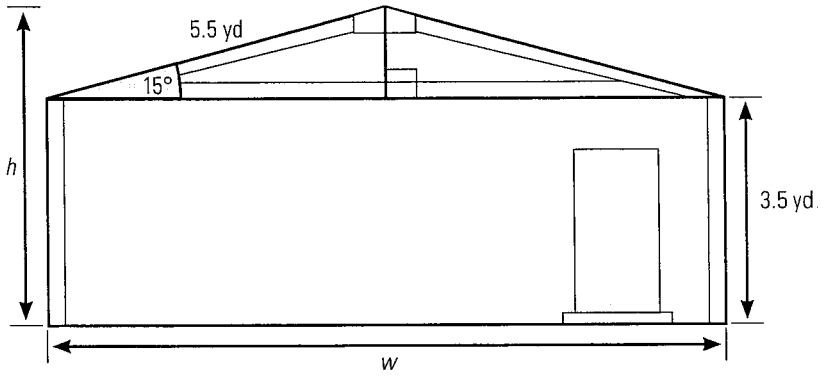
BUILD YOUR SKILLS

7. Solve the given triangle without using the Pythagorean theorem.



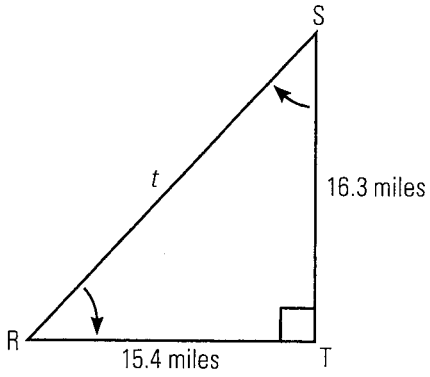
8. The two equal angles of an isosceles triangle are each 70°. Determine the measures of the rest of the triangle if it has a height of 16 cm.

9. The length of the rafter is 5.5 yards, and the side height of the building is 3.5 yards. Determine the width of the building and its total height.



Example 4

Solve the given triangle.



SOLUTION

Calculate $\angle R$ using the tangent ratio.

$$\tan R = \frac{\text{opp}}{\text{adj}}$$

$$\tan R = \frac{ST}{TR}$$

$$\tan R = \frac{16.3}{15.4}$$ Substitute the known values.

$$\angle R = \tan^{-1}\left(\frac{16.3}{15.4}\right)$$ Use the inverse function to solve for $\angle R$.

$$\angle R \approx 46.6263$$

$\angle R$ is approximately 47°.

Calculate $\angle S$ using the measures of the angles in the triangle.

$$\angle S = 180° - 90° - 47°$$

$$\angle S = 43°$$

You could have found $\angle S$ first using the tangent ratio.

Calculate the length of side t using the Pythagorean theorem.

$$r^2 + s^2 = t^2$$

$$16.3^2 + 15.4^2 = t^2$$

$$\sqrt{16.3^2 + 15.4^2} = t$$

$$\sqrt{265.69 + 237.16} = t$$

$$\sqrt{502.85} = t$$

$$22.42 \approx t$$

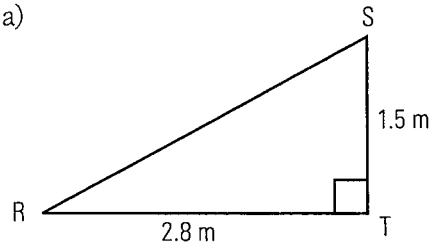
You could have solved for side t using the sine or cosine functions, but this would have involved rounding errors.

Side t is approximately 22.4 miles long.

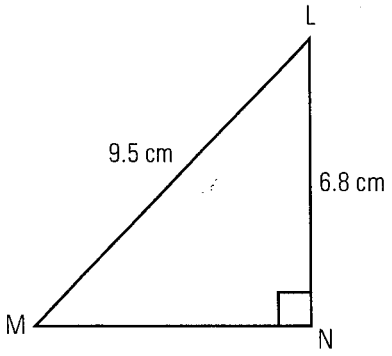
BUILD YOUR SKILLS

10. Solve the following triangles.

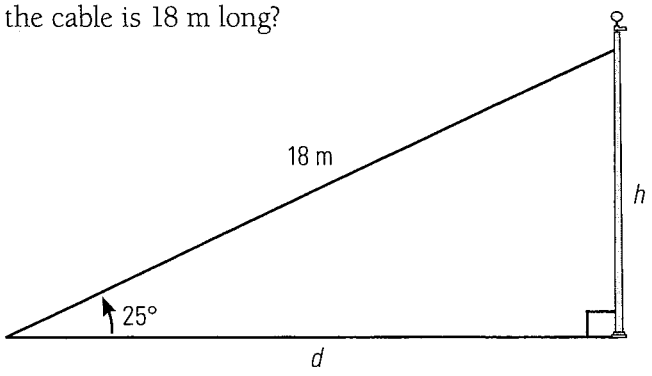
a)



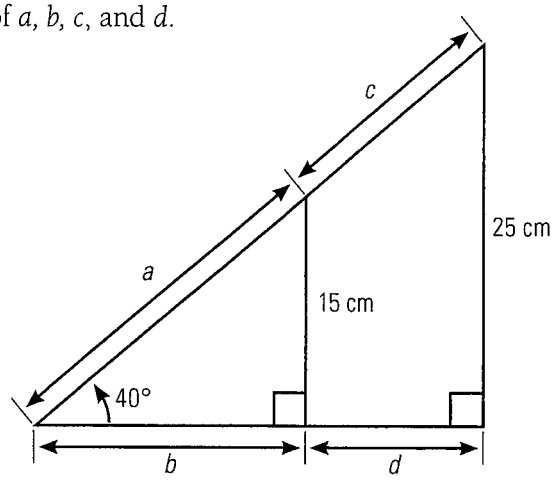
b)



11. What height is a pole, and how far away from it is a cable attached to the ground, if the angle of elevation is 25° and the cable is 18 m long?



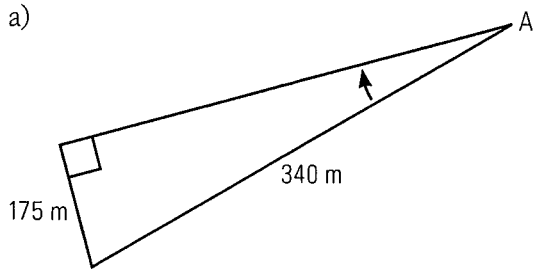
12. Find the values of a, b, c, and d.



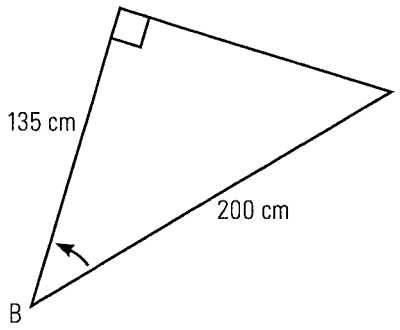
PRACTISE YOUR NEW SKILLS

1. Find the indicated angle in each of the following diagrams.

a)



b)



2. In a right triangle, one acute angle is 22° and the hypotenuse is 70 cm. Find the lengths of the legs and the other angle measure.

3. What is the angle of elevation if a ramp with a height of 1 metre and a horizontal length of 3 metres?

4. A grain auger is 25 feet long. The largest angle of elevation at which it can safely be used is 75°. What is the maximum height to which it can reach and how far from the base of the granary will it be, assuming that it dumps right at the edge?

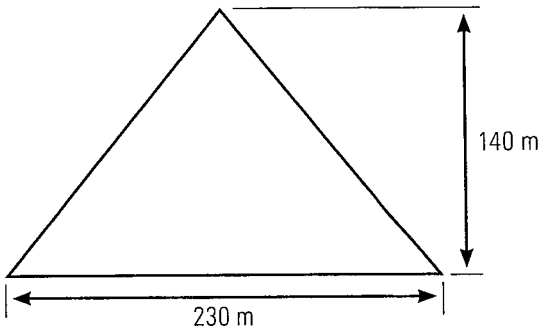
5. Maura's driveway has an angle of depression of 40° from the flat roadway. If it levels off to the garage floor, which is 3 metres below the roadway, how long is the driveway and how far into the lot is the garage entrance?

6. If a boat is 150 metres from the base of a cliff that is 90 metres high, what is the angle of elevation from the boat to the cliff top?

CHAPTER TEST

1. What is the length of a diagonal brace used to support a table that is 120 cm wide by 50 cm tall?

2. The Pyramid of Khufu is approximately 140 metres tall. If the base is a square with sides measuring 230 metres, what is the slant height from the centre of one of the sides of the pyramid? (Hint the slant height is the hypotenuse of a right triangle.)



3. A plane travels 12 km along its flight path while climbing at a constant rate of 8°. What is the vertical change in height during this time?

4. A ramp 12 metres long makes an angle of 15° with the ground. What is the height of the ramp? If the ramp is doubled in length, what will the total height be?

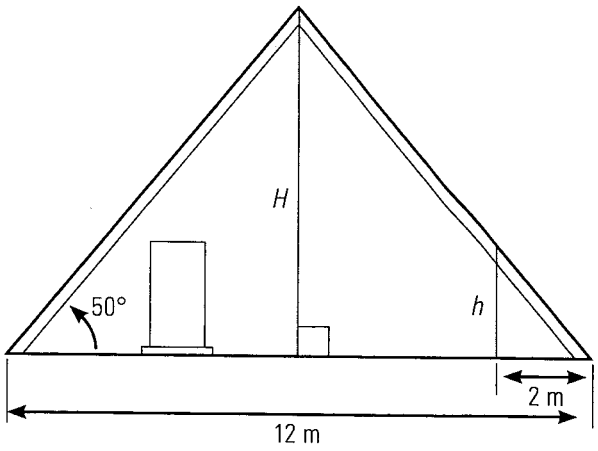
5. A chute from an open window to the ground makes an angle of 52° with the side of a building. If the window is 18 metres from the ground, how long is the chute?

6. A tree casts a shadow that is 10 metres long. If the angle of elevation to the top of the tree from the ground at the end of the shadow is 60°, how high is the tree?

7. The angle of elevation from the bottom of one building to the top of another building is 78°. The angle of elevation from the bottom of the second building to the top of the first is 62°. If the distance between them is 150 metres, how much taller is the higher building than the shorter one?

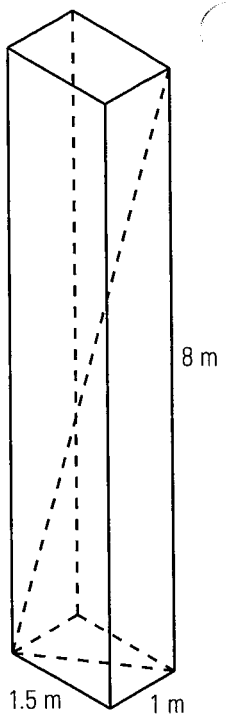
8. In an A-frame building, the angle of elevation of the roof is 50° and the building is 12 metres wide.

   a) How high is the building at the centre?



   b) How high is it 2 metres in from an edge?

9. A box is 1.5 m long, 1.0 m deep, and 8.0 m tall. What is the length of the longest object that can fit in the box?



10. A lifeguard sits in a chair that is 2.5 metres high. He spots a child in trouble in the water at an angle of depression of 23°. How far out from the chair is the child?

11. What is the angle of elevation of a playground slide that is 1.2 m high and has a horizontal length of 2.6 m?