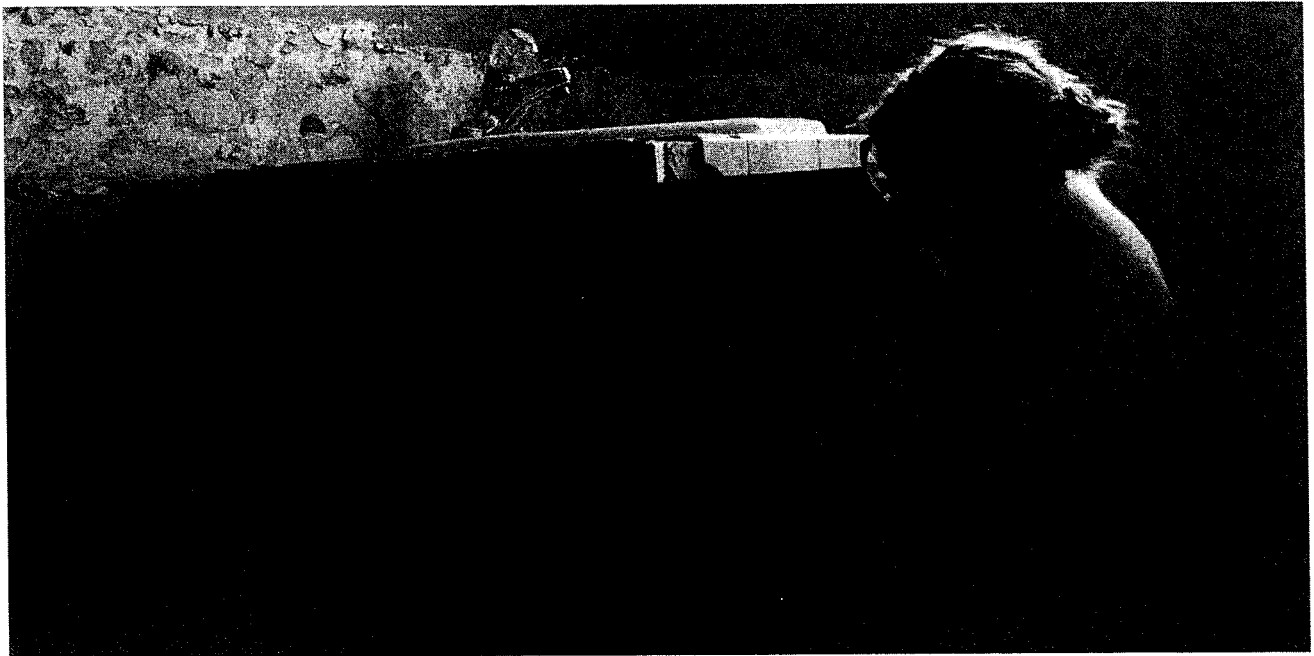


# Chapter 3

## Length, Area, and Volume



*Kristi Hansen is a Red Seal plumber. Calculating the capacity of water lines, determining the length of pipe needed for drainage systems, and accurately predicting the volume of hot water a building's system will use are some of her tasks.*

### 3.1

## Systems of Measurement

### REVIEW: WORKING WITH PERIMETER

**perimeter:** the sum of the lengths of all the sides of a polygon

In this section, you will calculate the **perimeter** of different shapes.

A square is a quadrilateral with 4 equal sides, so the perimeter can be found by the following formula:

$$P = 4 \times (\text{side length})$$

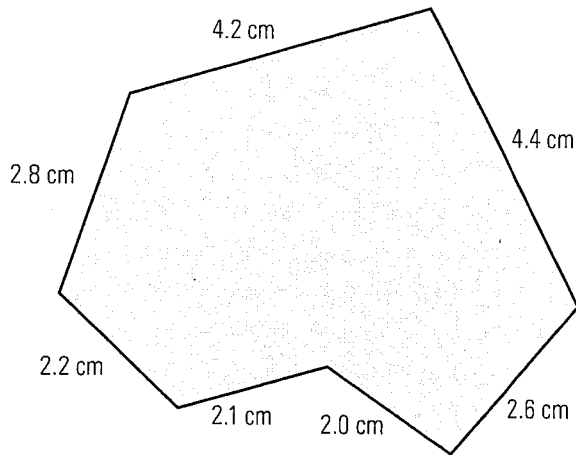
The perimeter of a rectangle with length  $\ell$  and width  $w$  can be found by the following formula:

$$P = 2\ell + 2w$$

$$P = 2(\ell + w)$$

**Example 1**

What is the perimeter of this figure?

**SOLUTION**

This figure is a heptagon, which means it has 7 sides. Its perimeter,  $P$ , is the sum of the lengths of all 7 sides.

$$P = 2.8 + 4.2 + 4.4 + 2.6 + 2.0 + 2.1 + 2.2$$

$$P = 20.3 \text{ cm}$$

The perimeter is 20.3 cm.

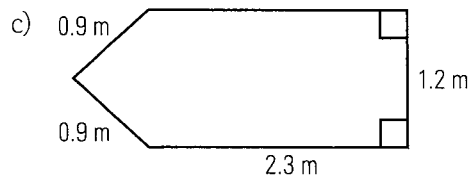
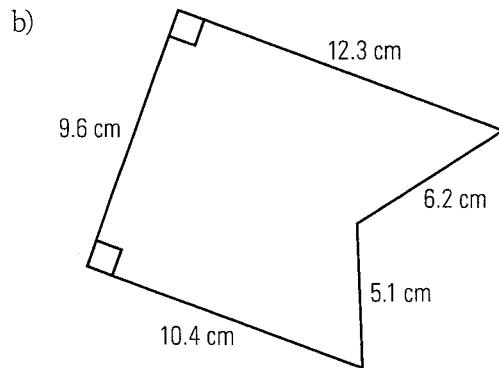
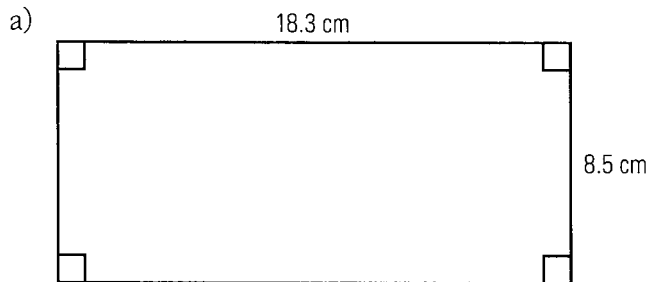
When the units of measurement are all the same, you can ignore them during calculations. Remember to add the units in at the end.

To make sure that you don't miss any sides when calculating the perimeter of a figure, start at one vertex and work your way around the figure.

**BUILD YOUR SKILLS**

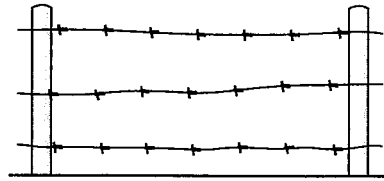
A small square symbol in the corner of a diagram means that it is a right angle.

1. Calculate the perimeters of the following diagrams.



2. Darma is edging a tablecloth with lace. The tablecloth is 210 cm by 180 cm. How much lace does she need?

3. Garry installs a wire fence around a rectangular pasture. The pasture measures 15 m by 25 m, and he uses three rows of barbed wire. How much wire did he use?

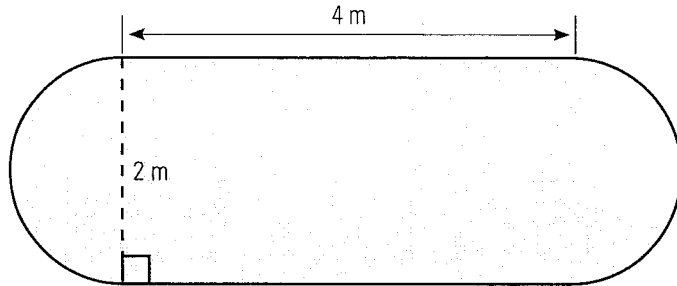


4. Chantal is building a fence around her swimming pool. The pool is 25 ft long and 12 ft wide, and she wants a 6-ft wide rectangular walkway around the entire pool. How much fencing will she need?

do 4 @ bottom first

**Example 2**

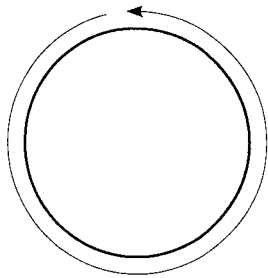
The sides of the flower garden shown below are 4 m long. Each end is a semi-circle with a diameter of 2 m: What is the perimeter of the flower garden?



**SOLUTION**

Break this problem down into two parts, a circle and a rectangle.

**circumference:** the measure of the perimeter a circle



If you add the two end sections together, they form a circle. You can use the formula for the **circumference** to find the perimeter:

$$C = \pi d \text{ or } 2\pi r$$

$C$  is the circumference,  $r$  is the radius,  $d$  is the diameter, and  $\pi$  is a constant. In this example, the diameter is 2 m.

Find the circumference of the ends of the flower garden by using this formula.

$$C = \pi d$$

$$C = \pi(2)$$

$$C \approx 6.28$$

Add the lengths of the two straight parts to the circumference of the circle to calculate the perimeter.

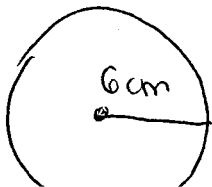
$$P \approx 6.28 + 4 + 4$$

$$P \approx 14.28$$

The perimeter of the flower garden is about 14.28 m.

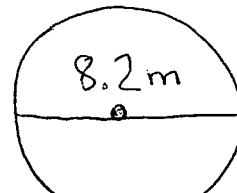
Try

a)



$2(3.14(6))$   
 $37.68 \text{ cm}$

b)

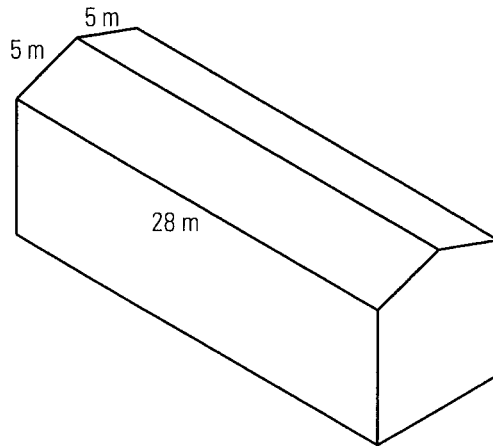


$3.14(8.2)$   
 $25.748 \text{ m}$

**BUILD YOUR SKILLS**

5. What is the circumference of a circular fountain if its radius is 5.3 m?

6. Johnny wants to put Christmas lights along the edge and peak of his roof. How many metres of lights will he need?



7. Hershey uses coloured wire to make a model of the Olympic symbol (5 interlocking circles). If each circle has a radius of 35 cm, how much wire does he need for the rings?

## NEW SKILLS: WORKING WITH SYSTEMS OF MEASUREMENT

**Système International (SI):** the modern version of the metric system; uses the metre as the basic unit of length

**imperial system:** the system most commonly used in the United States; the standard unit of measurement for length is the foot

Although there are other systems of measurement, the two most common are the **Système International (SI)** and the **imperial system**. In Canada, the official system of measurement is the SI. Because of Canada's close proximity to the United States, you should be familiar with both systems. Both are used in certain contexts.

Below are listed some common imperial units of length and their relationships.

$$12 \text{ inches (in or ")} = 1 \text{ foot (ft or ')}$$

$$36 \text{ inches} = 1 \text{ yard (yd)}$$

$$3 \text{ feet} = 1 \text{ yard}$$

$$5280 \text{ feet} = 1 \text{ mile (mi)}$$

$$1760 \text{ yards} = 1 \text{ mile}$$

For more details, see page 94 of *MathWorks 10*.

### Example 3

Wilhelmina, a seamstress, is sewing bridesmaids' dresses. She orders the fabric from the United States, where fabric is measured in yards. Each dress requires  $3\frac{3}{4}$  yards of silk,  $1\frac{1}{2}$  yards of lace fabric, and  $7\frac{1}{4}$  yards of trim. How much of each type of material does Wilhelmina need to make 5 dresses?

### SOLUTION

Multiply each amount by 5.

$$\text{silk} = 3\frac{3}{4} \times 5$$

$$\text{silk} = \frac{15}{4} \times 5$$

$$\text{silk} = \frac{75}{4}$$

$$\text{silk} = 18\frac{3}{4} \text{ yd}$$

Convert to an improper fraction and multiply.

Convert to a mixed fraction.

$$\text{lace fabric} = 1\frac{1}{2} \times 5$$

$$\text{lace fabric} = \frac{3}{2} \times 5$$

Convert to an improper fraction and multiply.

If you look at a ruler marked in imperial units, you will notice

it is usually divided into halves, quarters, eighths, and sixteenths, whereas the SI system uses tenths.

$$\text{lace fabric} = \frac{15}{2}$$

$$\text{lace fabric} = 7\frac{1}{2} \text{ yd} \quad \text{Convert to a mixed fraction.}$$

$$\text{trim} = 7\frac{1}{4} \times 5$$

$$\text{trim} = \frac{29}{4} \times 5 \quad \text{Convert to an improper fraction and multiply.}$$

$$\text{trim} = \frac{145}{4}$$

$$\text{trim} = 36\frac{1}{4} \text{ yd} \quad \text{Convert to a mixed fraction.}$$

Since fabric can be bought in partial yards, Wilhelmina will need to purchase  $18\frac{3}{4}$  yd of silk,  $7\frac{1}{2}$  yd of lace fabric, and  $36\frac{1}{4}$  yd of trim.

#### ALTERNATIVE SOLUTION

Convert each mixed fraction to a decimal, and then multiply by 5.

$$\text{silk} = 3\frac{3}{4}$$

$$\text{silk} = 3.75$$

$$\text{silk} = 3.75 \times 5$$

$$\text{silk} = 18.75$$

$$\text{lace fabric} = 1\frac{1}{2}$$

$$\text{lace fabric} = 1.5$$

$$\text{lace fabric} = 1.5 \times 5$$

$$\text{lace fabric} = 7.5$$

$$\text{trim} = 7\frac{1}{4}$$

$$\text{trim} = 7.25$$

$$\text{trim} = 7.25 \times 5$$

$$\text{trim} = 36.25$$

Since fabric can be bought in partial yards, Wilhelmina will need to purchase 18.75 yd of silk, 7.5 yd of lace fabric, and 36.25 yd of trim.



**BUILD YOUR SKILLS**

A 2 by 4 is not exactly 2" by 4".

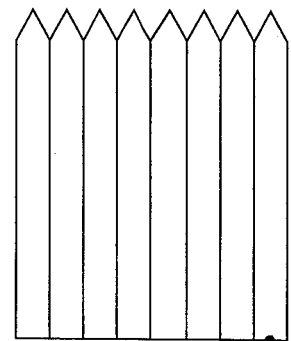
The name comes from the dimensions of the lumber before it is dried; when the lumber dries, it shrinks and then is replaned to make it a standard size.

A 2 by 4 is actually  $1\frac{1}{2}$ " by  $3\frac{1}{2}$ ".

Lumber and other building supplies are usually sold using imperial units.

8. Bernard is buying some lumber to finish a project. He needs 3 pieces of 2 by 4 that are each  $4\frac{1}{2}$  feet long, and 10 pieces of 2 by 2 that are each  $5\frac{1}{4}$  feet long. How much of each does he need in total?
9. Benjie is replacing some plumbing pipes. He needs 3 pieces of copper pipe: one piece is 2 feet long, one is 5 feet 7 inches long, and one is 4 feet long. How much copper pipe does he need if he loses 1 inch when he cuts the pipe and he can only buy it in even numbers of feet?

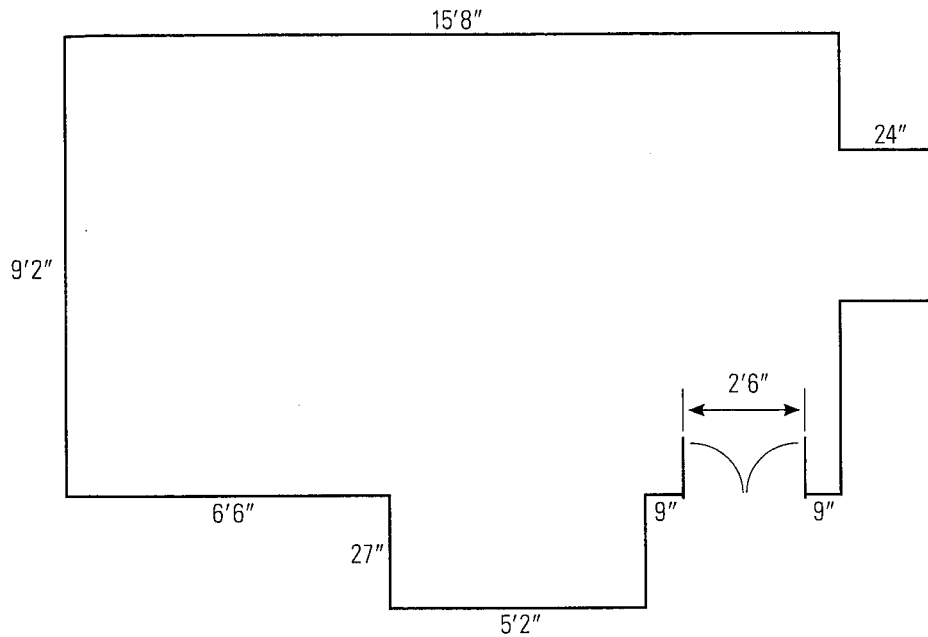
10. If each board in a fence is 6 inches wide, how many of them will José need to fence a playground that is 60 feet wide by 125 feet long?



board is 6" wide

**Example 4**

Fatima is trying to calculate how much baseboard she will need for the room shown below.



What is the minimum amount of baseboard she will need?

**SOLUTION**

Find the perimeter of the room. Since there is a door, no baseboard will be needed there. Measurements are given in feet and/or inches.

To find the perimeter of the room, start at any one point, such as the edge of the door, and work your way around the room.

$$P = 9'' + 27'' + 5'2'' + 27'' + 6'6'' + 9'2'' + 15'8'' + 9'2'' + 24'' + 24'' + 9''$$

Add feet to feet and inches to inches to get  $44' 140''$ .

Convert 140 inches to feet by dividing by 12 (because 12 inches equals 1 foot).

$$\frac{140}{12} = 11 \text{ remainder } 8, \text{ or } 11'8''$$

Add this to the measure in feet.

$$44' + 11'8'' = 55'8''$$

Therefore, she needs  $55'8''$  of baseboard.

Where did the second  $9'2''$  come from?

**ALTERNATIVE SOLUTION**

Combine the lengths of the smaller wall segments to simplify the calculation, and subtract the width of the door.

$$P = 2(15'8") + 2(9'2") + 2(27") + 2(24") - 2'6"$$

$$P = 30'16" + 18'4" + 54" + 48" - 2'6"$$

$$P = 46'116"$$

Convert 116 inches to feet.

$$\frac{116}{12} = 9, \text{ remainder } 8, \text{ or } 9'8"$$

Add this to the measure in feet.

$$46' + 9'8" = 55'8"$$

Therefore, she needs 55' 8" of baseboard.

**BUILD YOUR SKILLS**

11. A pet shop stores 5 pet cages that are 2'8" wide, 3 cages that are 4'6" wide, and 2 cages that are 1'8" wide. Can these cages fit side by side along a wall that is 30' long?
  
  
  
  
  
  
  
  
  
  
12. A circular garden is 6'4" in diameter. To plant a geranium approximately every foot along the circumference, how many geraniums are needed?

13. The height of a basement ceiling is  $7'2''$ . A  $6''$ -deep heating pipe runs across the middle. To enclose it, there must be a 1-inch space between the pipe and the drywall. Will Craig, who is  $6'6''$  tall, be able to walk under the finished pipe?

**PRACTISE YOUR NEW SKILLS**

1. Convert the following measurements.

a) 42 inches to feet

b) 16 inches to feet and inches

c) 96 inches to yards

d) 5 miles to yards

2. You are building a fence around your vegetable garden in your backyard. If the garden is  $12'8''$  long and  $4'6''$  wide, what is the total length of fencing you will need?
3. Marjorie is building a dog run that is  $25'8''$  long and  $8'8''$  wide. How much fencing will she need if the opening is  $3'6''$  wide and will not need fencing?
4. A package of paper is  $2''$  high and  $8.5''$  wide. If a warehouse shelf is  $1'5''$  high and 2 yards long, how many packages of paper can be put on the shelf?
5. Jennine estimates that each step she takes is  $18''$  long and that she takes 1550 steps per block. How many blocks must she walk if she wants to walk 5 miles?

# Converting Measurements

32

## REVIEW: WORKING WITH VARIABLES WITHIN FORMULAS

In this section, you will practise substituting known values into formulas.

In this chapter, you will need to use the following formulas.

$$A = \ell w \quad \text{Area of a rectangle, where } \ell \text{ is the length and } w \text{ is the width.}$$

$$A = \pi r^2 \quad \text{Area of a circle, where } r \text{ is the radius and } \pi \text{ is the constant, pi.}$$

$$A = \frac{1}{2}bh \quad \text{Area of a triangle, where } b \text{ is the length of the base and } h \text{ is the height.}$$

$$A = \pi rs \quad \text{Area of the surface of a cone, where } r \text{ is the radius and } s \text{ is the slant height.}$$

$$C = 2\pi r \quad \text{Circumference of a circle, where } r \text{ is the radius.}$$

### Example 1

*Sumo* is a traditional Japanese martial art. The area of a circular sumo ring, or *dohyo*, is  $16.26 \text{ m}^2$ . What is the radius of the ring?

#### SOLUTION

Use the formula for finding the area of a circle. You are given the area of the circle, so substitute it into the formula and solve for the unknown value,  $r$ .

$$A = \pi r^2$$

$$16.26 = \pi r^2$$

$$\frac{16.26}{\pi} = \frac{\cancel{\pi} r^2}{\cancel{\pi}} \quad \text{Divide both sides by } \pi \text{ to isolate } r^2.$$

$$\sqrt{\frac{16.26}{\pi}} = r$$

$$2.28 \approx r$$

The radius of a sumo ring is 2.28 m.

**BUILD YOUR SKILLS**

1. Ina is laying turf in a yard measuring 38 ft by 20 ft. What is the yard's area in square feet?
2. A store advertises a circular rug as being  $4.9 \text{ m}^2$ . Travis wants a rug to fit a rectangular space that is 2.6 m by 2.6 m. Will this rug fit?
3. You are designing a rectangular label for canned food. The can is 5 cm high, with a diameter of 9 cm. To plan your design, calculate the label's length. (The length is equal to the circle's circumference.)

**NEW SKILLS: WORKING WITH DIFFERENT SYSTEMS OF MEASUREMENT**

The official system of measurement in Canada is the SI, but the United States uses imperial units. If you are buying products from the United States or are doing business with a US company, you will need to convert between the two systems of measurement.

Below are some common relationships between SI and imperial units of length.

$$1 \text{ inch} \approx 2.54 \text{ centimetres}$$

$$1 \text{ foot} \approx 0.3 \text{ metres}$$

$$1 \text{ yard} \approx 0.9 \text{ metres}$$

$$1 \text{ mile} \approx 1.6 \text{ kilometres}$$

For more details, see page 106 of *MathWorks 10*.

**Example 2**

Mary is delivering a load of goods from Vancouver, BC, to Seattle, WA, then in Seattle, she is picking up another load to deliver to Albuquerque, NM. The distance from Vancouver to Seattle is 220 km and the distance from Seattle to Albuquerque is 1456 mi. The odometer in Mary's truck records distance in kilometres.

- What is the total distance she will travel, in kilometres?
- If her odometer read 154 987 km when she left Seattle, what did it read when she left Vancouver?
- What will her odometer read when she reaches Albuquerque?

**SOLUTION**

- Find the distance in kilometres from Seattle to Albuquerque.

$$1 \text{ mi} \approx 1.6 \text{ km}$$

$$1456 \text{ mi} \approx 1456 \times 1.6$$

$$1456 \text{ mi} \approx 2330 \text{ km}$$

The distance from Seattle to Albuquerque is 2330 km.

Add this to the distance from Vancouver to Seattle to find the total distance.

$$220 \text{ km} + 2330 \text{ km} = 2550 \text{ km}$$

Her trip will be about 2550 km.

- Subtract the distance she travelled from the odometer reading.

$$154\,987 - 220 = 154\,767$$

Her odometer read 154 767 when she left Vancouver

- Add the distance from Seattle to Albuquerque to the odometer reading.

$$154\,987 + 2330 = 157\,317$$

Her odometer should read about 157 317 when she reaches Albuquerque.



**BUILD YOUR SKILLS**

4. Suzanne purchased tiles for her patio that are 8" by 4". She measured her patio in metres and wants to convert the tile dimensions to SI units. What are the dimensions of the tiles in centimetres?
  
  
  
  
  
  
  
  
  
  
5. Benjamin owns an older American truck. The odometer shows distance travelled in miles. On a recent trip to deliver produce for his employer, he drove 1564 mi. His employer pays him \$0.89/km for the use of his own truck. How much will he be reimbursed for the use of his truck for the trip?

6. Marnie owns a carpet store and sells hallway runners for \$9.52/linear foot.

a) How much is this per linear yard?

28.56

b) How much is this per linear metre?

31.73

c) Ralph needs 3.9 m of the runner for his hallway. How much will it cost?

123.75

**Example 3**

Rebecca is planning to install sod in her backyard, which is 18.2 m by 9.8 m. If sod costs  $\$0.28/\text{ft}^2$ , how much will it cost to sod the backyard?

**SOLUTION**

Change the measurements of the backyard to feet, and then find the area.

$$0.3 \text{ m} = 1 \text{ ft}$$

$$18.2 \text{ m} = \frac{18.2}{0.3} \text{ ft}$$

$$\frac{18.2}{0.3} \approx 60.7 \text{ ft}$$

Her yard is approximately 61 ft long.

Similarly, change 9.8 m (980 cm) to feet.

$$1 \text{ m} = \frac{1}{0.3} \text{ ft}$$

$$9.8 \text{ m} = \frac{9.8}{0.3} \text{ ft}$$

$$\frac{9.8}{0.3} \approx 32.7 \text{ ft}$$

Her yard is approximately 33 ft wide.

Calculate the area of her backyard. The area of a rectangle is calculated by multiplying the length by the width.

$$A = \ell w$$

$$A = 61 \times 33$$

$$A = 2013 \text{ ft}^2$$

She will need approximately 2013 square feet of sod at  $\$0.28/\text{ft}^2$ .

$$2013 \times \$0.28 = \$563.64$$

It will cost about  $\$563.64$  to sod her backyard.

**BUILD YOUR SKILLS**

7. You could have solved Example 3 by determining the cost of the sod per square metre. Answer the question using this method. Is your answer the same? Why or why not?
8. Kuldeep has been hired to lay terracotta tiles on a floor that measures 4.2 m by 3.8 m. The tiles are 9" by 9" and come in boxes of 12.
- a) How many boxes must he buy? (He cannot buy a partial box.)
- b) If the tiles cost \$18.95 per box, how much will the tiles cost in total?

9. Toula calculates the cost of cementing the bottom and sides of a circular pond. When all costs are considered, the job will cost \$175.85 per square metre of finished area. If the pond has a radius of 3 feet and a depth of 2 feet, how much will she charge for the job?

**PRACTISE YOUR NEW SKILLS**

- Juan is a picture framer. He is framing a picture that is 24 inches by 32 inches with a frame that is 2.5 inches wide. What is the outer perimeter of the framed picture:
  - in inches?
  - in feet and inches?
  - in yards, feet, and inches?
- Charlie drove from Calgary to Saskatoon, which is a distance of 620 km. How far is this in miles?

3. A school custodian must mark off a field that is 150 ft by 85 ft. His tape measure is marked in metres. What are the dimensions of the field in metres (to the nearest tenth of a metre)?
4. Jeff knows that his semi-trailer truck is 3.2 m high. A tunnel is marked as "Max height: 10'6". Will Jeff's truck fit through the tunnel?
5. Carla needs 3.5 m of cloth. If the cloth she wants to purchase costs \$9.78/yd, how much will the cloth cost?

6. Ari, a gardener, estimates the cost of seeding a 150 m by 210 m area with grass seed. He needs 3 pounds of seed per 100 000 square feet. How many pounds of seed will Ari need?
7. A room measures 12'8" by 10'9". Carpeting costs \$45.98/m<sup>2</sup>. A customer will have to purchase 10% more carpeting than floor area due to waste and he cannot purchase partial square metres. What is the minimum cost of the carpeting?



## 3.3

## Surface Area

## NEW SKILLS: WORKING WITH SURFACE AREA

**surface area:** the total area of all the faces, or surfaces, of a three-dimensional object; measured in square units

**net:** a two-dimensional pattern used to construct three-dimensional shapes

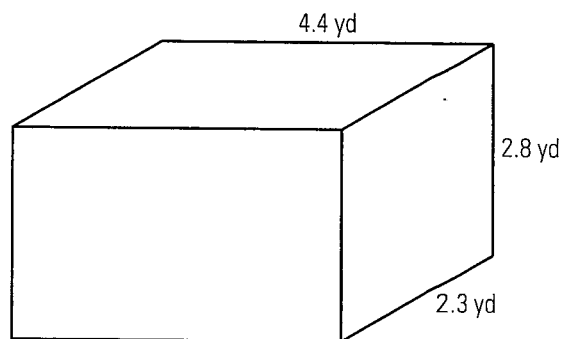
**Surface area** is the area that would be covered by a three-dimensional (3-D) object if you could lay it out flat. A **net** is a diagram of a 3-D object seen as a flat surface.

If you know how to find the area of 2-D shapes, you can find the surface area of 3-D objects by breaking them down to their component surfaces and adding the areas together.

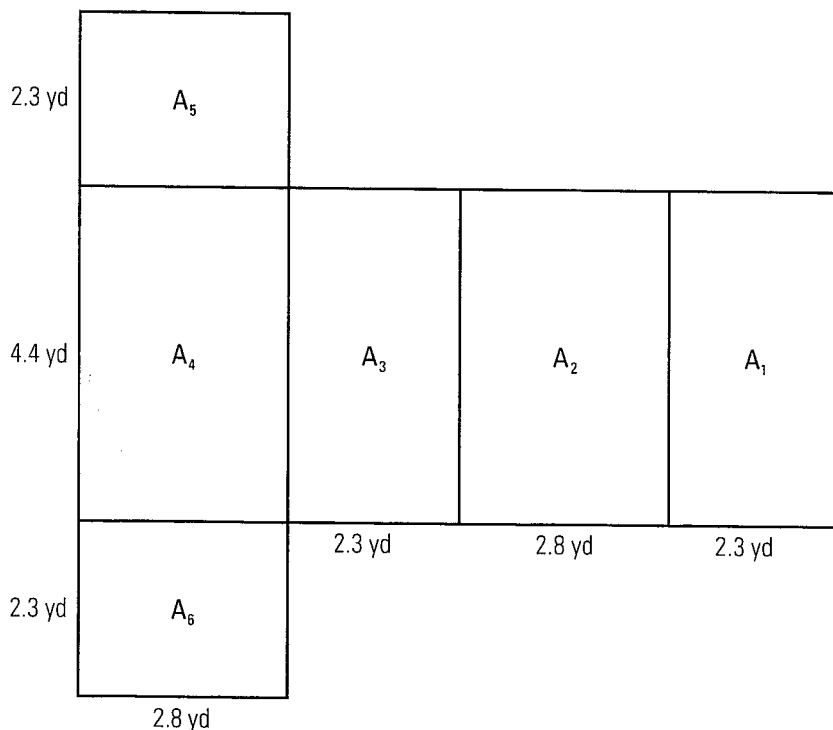
For more details, see page 115 of *MathWorks 10*.

**Example 1**

Akiko has been hired to paint the exterior of a storage bin. If the bin is a rectangular prism that measures 2.3 yards by 4.4 yards by 2.8 yards, what is the surface area of the bin?

**SOLUTION**

Sketch a net of the bin.



Calculate the area of each of the rectangles and add together to find the surface area (SA).

$$A_1 = \ell w$$

$$A_1 = 4.4 \times 2.3$$

$$A_1 = 10.12 \text{ yd}^2$$

The area of  $A_3$  is the same as  $A_1$ .

$$A_2 = \ell w$$

$$A_2 = 4.4 \times 2.8$$

$$A_2 = 12.32 \text{ yd}^2$$

The area of  $A_4$  is the same as  $A_2$ .

$$A_5 = \ell w$$

$$A_5 = 2.3 \times 2.8$$

$$A_5 = 6.44 \text{ yd}^2$$

The area of  $A_6$  is the same as  $A_5$ .

Total surface area:

$$SA = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$$

$$SA = 10.12 + 12.32 + 10.12 + 12.32 + 6.44 + 6.44$$

$$SA = 57.76 \text{ yd}^2$$

The bin has a surface area of  $57.76 \text{ yd}^2$ .

**ALTERNATIVE SOLUTION**

Many people prefer to measure the different surfaces of the object.

There are:

2 rectangles that are 4.4 yd by 2.8 yd (front and back)

2 rectangles that are 4.4 yd by 2.3 yd (top and bottom)

2 rectangles that are 2.3 yd by 2.8 yd (ends)

Find the area of each rectangle and then add.

$$A_{1+3} = 2 \times 4.4 \text{ yd} \times 2.3 \text{ yd}$$

$$A_{1+3} = 20.24 \text{ yd}^2$$

$$A_{2+4} = 2 \times 4.4 \text{ yd} \times 2.8 \text{ yd}$$

$$A_{2+4} = 24.64 \text{ yd}^2$$

$$A_{5+6} = 2 \times 2.3 \text{ yd} \times 2.8 \text{ yd}$$

$$A_{5+6} = 12.88 \text{ yd}^2$$

$$SA = A_{1+3} + A_{2+4} + A_{5+6}$$

$$SA = 24.64 \text{ yd}^2 + 20.24 \text{ yd}^2 + 12.88 \text{ yd}^2$$

$$SA = 57.76 \text{ yd}^2$$

The bin has a surface area of 57.76 yd<sup>2</sup>.

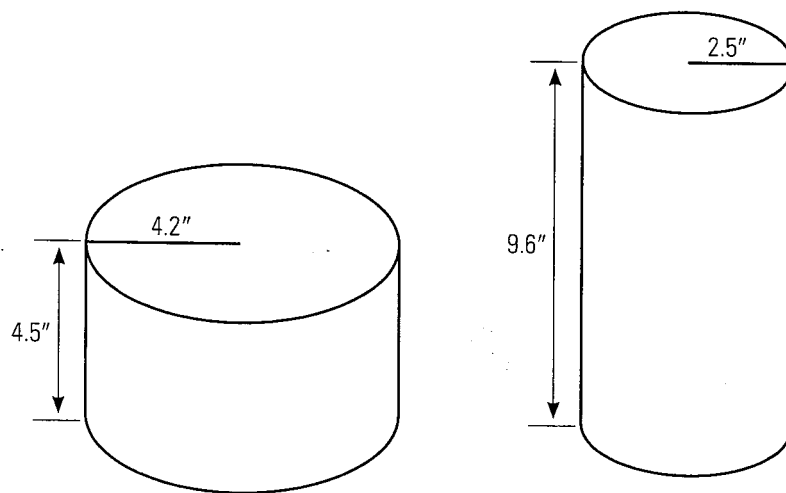
**BUILD YOUR SKILLS**

1. Jim has been hired to make a jewellery box. If the box is 12 inches long, 6 inches deep, and 9 inches tall, how much veneer will it take to cover the exterior, assuming no waste?
2. Anita is building a greenhouse onto the side of her garage. She wants it to be 6 feet long, 4 feet wide, and 3 feet high, with the 6-foot long side against the side of her garage. What area of glass will she need to complete the greenhouse? (Hint: No glass will be used along the side of the garage or for the floor.)
3. Vicki is tiling her 35" by 35" shower stall. The tiles reach the 8-foot ceiling on 3 sides. How many square inches of tiles should she purchase to tile the walls and floor?

**Example 2**

A canning factory wants to use as little metal as possible to make its cans. It considers two can sizes that each hold about the same amount. One is 4.5 inches tall with a radius of 4.2 inches and another is 9.6 inches tall with a radius of 2.8 inches.

Which can should they use? Why?

**SOLUTION**

To find out which can uses the least amount of metal, calculate the surface area of each can.

The top and bottom of the cans are circles. The area of a circle can be found by the following formula:

$$A = \pi r^2$$

If the side of the can is rolled out flat, it will form a rectangle, where the width is the height of the can and the length is the circumference of the circle.

**Can #1**

Top and bottom:

$$A = 2\pi r^2$$

$$A = 2\pi(4.2)^2$$

$$A \approx 110.8 \text{ in}^2$$

Side:

$$A = 2\pi rh$$

$$A = 2\pi(4.2)(4.5)$$

$$A \approx 118.7 \text{ in}^2$$

$$SA = A_{\text{top + bottom}} + A_{\text{side}}$$

$$SA \approx 110.8 + 118.7$$

$$SA \approx 229.5 \text{ in}^2$$

**Can #2**

Top and bottom:

$$A = 2\pi r^2$$

$$A = 2\pi(2.8)^2$$

$$A \approx 49.2 \text{ in}^2$$

Side:

$$A = 2\pi rh$$

$$A = 2\pi(2.8)(9.6)$$

$$A \approx 168.8 \text{ in}^2$$

$$SA = A_{\text{top + bottom}} + A_{\text{side}}$$

$$SA \approx 49.2 + 168.8$$

$$SA \approx 218 \text{ in}^2$$

Considering surface area, they should use the second can because it uses less material.

**BUILD YOUR SKILLS**

4. A cylindrical shipping tube is 48 inches tall and 6 inches in diameter.

a) What is its surface area in square inches?

b) What is its surface area in square feet?

5. Sanjiv designs a cylindrical box to hold 4 tennis balls stacked one on top of the other. If a tennis ball is approximately  $3\frac{1}{4}$  inches in diameter, what is the surface area of the box? (Ignore the thickness of the material.)

6. Jennifer must make a conical funnel out of sheet metal. If the funnel is 9 inches tall, has a slant height of 10.7 inches, and has a radius of 5.8 inches at the top, what is the surface area of the sheet metal in square feet?

**Example 3**

Harry has to paint the walls and ceiling of a room that is 12 ft long, 10 ft wide, and  $8\frac{1}{2}$  ft high. There is a 6 ft by 4 ft window, a  $2\frac{1}{2}$  ft by 7 ft doorway, and a mirrored closet door that is 6 ft by 7 ft. What surface area must he paint?

**SOLUTION**

Find the total area of the four walls and the ceiling and subtract the areas of the window and doors.

Area of ceiling:

$$A_1 = 12 \text{ ft} \times 10 \text{ ft}$$

$$A_1 = 120 \text{ ft}^2$$

Area of long walls:

$$A_2 = 12 \text{ ft} \times 8\frac{1}{2} \text{ ft} \times 2$$

$$A_2 = 204 \text{ ft}^2$$

Area of shorter walls:

$$A_3 = 10 \text{ ft} \times 8\frac{1}{2} \text{ ft} \times 2$$

$$A_3 = 170 \text{ ft}^2$$

Area of walls and ceiling, including openings:

$$A_4 = A_1 + A_2 + A_3$$

$$A_4 = 120 \text{ ft}^2 + 204 \text{ ft}^2 + 170 \text{ ft}^2$$

$$A_4 = 494 \text{ ft}^2$$

Calculate the areas of the window, door, and closet that will not be painted.

Area of window:

$$A_5 = 6 \text{ ft} \times 4 \text{ ft}$$

$$A_5 = 24 \text{ ft}^2$$

Area of door:

$$A_6 = 2\frac{1}{2} \text{ ft} \times 7 \text{ ft}$$

$$A_6 = 17\frac{1}{2} \text{ ft}^2$$



Area of closet door:

$$A_7 = 6 \text{ ft} \times 7 \text{ ft}$$

$$A_7 = 42 \text{ ft}^2$$

Total area that does not need painting:

$$A_8 = A_5 + A_6 + A_7$$

$$A_8 = 24 \text{ ft}^2 + 17 \frac{1}{2} \text{ ft}^2 + 42 \text{ ft}^2$$

$$A_8 = 83 \frac{1}{2} \text{ ft}^2$$

Total area to be painted:

$$A_{total} = A_4 - A_8$$

$$A_{total} = 494 \text{ ft}^2 - 83 \frac{1}{2} \text{ ft}^2$$

$$A_{total} = 410 \frac{1}{2} \text{ ft}^2$$

The total area to be painted is  $410 \frac{1}{2} \text{ ft}^2$ .

### **BUILD YOUR SKILLS**

7. Geneviève plans to apply two coats of paint to the walls of her garden shed. The shed is 8 feet long by 6 feet wide by 7 feet tall. If there are 3 windows that are 2 feet by 18 inches each, what will be the total area she paints?

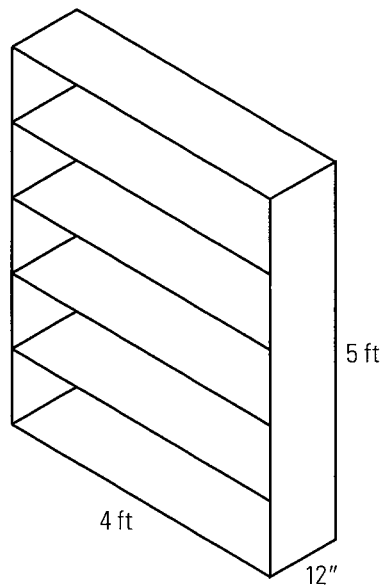
8. A metal cylindrical canister is  $1'3''$  long and has a diameter of 4 inches. What is the total surface area of the cylinder?
9. Jerg is finishing the sides and bottom of a hot tub. The hot tub is  $6'11''$  long,  $5'6''$  wide, and 3' deep. It has a bench going all the way around the inside that is 16" high and 15" wide that will not be finished. How much finishing material will Jerg need?

**Example 4**

The wood that Terrance wants to use to make a shelving unit costs \$6.49/ft<sup>2</sup>. How much will it cost him (assuming no wastage) to make a shelving unit that is 4 ft wide by 12 inches deep by 5 ft tall if there are 4 shelves (plus the top and bottom)?

**SOLUTION**

Sketch the unit.



There will be 6 pieces (shelves) that are 12" (or 1') by 4'.

There will be 2 pieces (ends) that are 1' by 5'.

There will be 1 piece (back) that is 4' by 5'.

Find the areas of each piece and add.

$$A_1 = 6 \times 1 \text{ ft} \times 4 \text{ ft}$$

$$A_1 = 24 \text{ ft}^2$$

$$A_2 = 2 \times 1 \text{ ft} \times 5 \text{ ft}$$

$$A_2 = 10 \text{ ft}^2$$

$$A_3 = 1 \times 4 \text{ ft} \times 5 \text{ ft}$$

$$A_3 = 20 \text{ ft}^2$$

$$\text{Total area} = 24 \text{ ft}^2 + 10 \text{ ft}^2 + 20 \text{ ft}^2$$

$$\text{Total area} = 54 \text{ ft}^2$$

Cost:

$$\$6.49 \times 54 = \$350.46$$

The total cost of the wood will be \$350.46.

**BUILD YOUR SKILLS**

10. Randi is installing flooring in her den. The room is 12 feet by 19 feet. A fireplace that is 6 feet wide juts out 2 feet into the room. Also, there are 2 built-in bookcases, each 1 foot deep and 3 feet wide. She needs to order 12% more flooring than required because of wastage and cutting. How much will it cost if the wood costs \$5.25/ft<sup>2</sup>?
11. Sheet metal costs \$54.25/yd<sup>2</sup>. How much will it cost Hamish to cover a conical roof if it has a radius of 2.2 yards and a slant height of 3.5 yards?

12. Ted is wallpapering his bedroom. The room is 10 ft long by 8 ft wide by  $9\frac{1}{2}$  ft tall. There is a door measuring 3 ft by 7 ft and a window measuring 3 ft by 4 ft. Each double roll covers approximately  $56 \text{ ft}^2$  and costs \$29.95. How much will the wallpaper cost?

**PRACTISE YOUR NEW SKILLS**

1. A storage bin is a rectangular prism that measuring 114 cm long by 56 cm high by 56 cm deep. What is the surface area of the exterior, in square feet?
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
2. How much glass is needed to construct a hexagonal (6-sided) fish tank? The tank is 4 feet tall and each pane is  $1\frac{1}{2}$  feet wide. (The tank's bottom is not made of glass.)

3. Stan is deciding between patio tiles that are 39 cm by 39 cm and tiles that are 18 cm by 27 cm. His patio is 3 m by 2.5 m. Considering area only, how many of each type of tile would he need?
4. A chocolate bar is in the shape of a triangular prism. The box is  $8\frac{1}{2}$  inches long and the ends are equilateral triangles with sides measuring  $1\frac{3}{4}$  inches. What is the surface area of the box? (Hint: You need to find the height of the triangle. Use the following formula,  $A = \frac{1}{2}bh$ )

5. Jocelyne is designing a logo for the outside of a cylindrical water bottle. She knows that the bottle is  $7\frac{1}{2}$  in tall and has a diameter of  $3\frac{3}{4}$  in. How large can the label on the water bottle be?
6. A conical paper cup has a slant height of  $3\frac{1}{8}$  inches and a diameter of 3 inches. How much paper is needed to make the cup?
7. The Great Pyramid of Giza has a square base that, when built, was approximately 756 ft on each side. If the slant height of each triangular side was approximately 610 ft, what was the original surface area of the pyramid?



## 3.4

## Volume

**NEW SKILLS: WORKING WITH VOLUME**

**volume:** the amount of space an object occupies

The **volume** of a solid is a measure of how much space it occupies. Volume is measured in cubic units.

The volume of a prism is calculated using the following formula.

$$V = A_{\text{base}} \times h$$

Volume is the product of the area of the base times the height of the object.

For more details, see page 124 of *MathWorks 10*.

**Example 1**

Alfred has a bulk container that holds 2000 cubic inches of dog biscuits. He plans to sell the biscuits in small boxes that measure 5" by 8" by 6". How many boxes will he need to sell all the dog biscuits?

**SOLUTION**

Find the volume of the small box.

$$V = \ell wh$$

$$V = 5'' \times 8'' \times 6''$$

$$V = 240 \text{ cu in}$$

Divide 2000 by 240.

$$2000 \div 240 = 8.3$$

Round up, so that all of the biscuits fit in boxes. Alfred would need 9 small boxes.

**BUILD YOUR SKILLS**

1. A fish tank is a rectangular prism that is 30 inches long, 24 inches deep, and 18 inches high. How much water will it hold:

  - a) in cubic inches?
  
  
  
  
  
  
  
  
  
  
  - b) in cubic feet?
  
2. Petra must stack boxes that are 3 ft by  $2\frac{1}{2}$  ft by  $1\frac{1}{2}$  ft onto a truck. What is the volume of each box?
  
  
  
  
  
  
  
  
  
  
3. Will the contents of a box that is 3 inches by 4 inches by 6 inches fit into a cube with sides of 4 inches?

### Example 2

Paulino runs a landscaping business. He needs to cover an area that is 10.8 m by 9.5 m with 10 cm of topsoil. How much will it cost if the soil costs \$18.75/yd<sup>3</sup>, and soil is available in multiples of  $\frac{1}{2}$  yd<sup>3</sup>?

#### SOLUTION

Find the volume of topsoil needed.

$$V = \ell wh$$

$$V = 10.8 \text{ m} \times 9.5 \text{ m} \times 0.1 \text{ m}$$

$$V = 10.26 \text{ m}^3$$

Calculate how many yards are in 1 metre.

$$1 \text{ cm} = \frac{1}{2.54} \text{ in}$$

$$100 \text{ cm} = \frac{100}{2.54} \text{ in}$$

$$100 \text{ cm} = 39.37 \text{ in}$$

$$1 \text{ m} = 39.37 \text{ in}$$

$$1 \text{ in} = \frac{1}{36} \text{ yd}$$

$$39.37 \text{ in} = \frac{39.37}{36} \text{ yd}$$

$$1 \text{ m} \approx 1.09 \text{ yd}$$

Calculate how many cubic yards are in a cubic metre by cubing both sides.

$$1 \text{ m}^3 = 1.09 \text{ yd}^3$$

$$1 \text{ m}^3 \approx 1.3 \text{ yd}^3$$

Since he needs 10.26 cubic metres, change cubic metres to cubic yards.

$$10.26 \text{ m}^3 = 10.26 \text{ m}^3 \times \frac{1.3 \text{ yd}^3}{1 \text{ m}^3}$$

$$10.26 \text{ m}^3 \approx 13.3 \text{ yd}^3$$

Remember:

$$10 \text{ cm} = 0.1 \text{ m}$$

He needs  $13.3 \text{ yd}^3$ , but Paulino will have to round up to the nearest  $\frac{1}{2} \text{ yd}^3$ , to  $13.5 \text{ yd}^3$ . Since each cubic yard costs \$18.75, multiply the number of cubic yards by the cost per cubic yard.

$$13.5 \times \$18.75 = \$253.13$$

The topsoil will cost \$253.13.

### ALTERNATIVE SOLUTION

Begin by changing the measurements to yards.

Using the calculations above, 1 m is approximately 1.09 yd.

$$10.8 \text{ m} = 10.8 \times 1.09 \text{ yd}$$

$$10.8 \text{ m} \approx 11.8 \text{ yd}$$

$$9.5 \text{ m} = 9.5 \times 1.09 \text{ yd}$$

$$9.5 \text{ m} \approx 10.4 \text{ yd}$$

$$0.1 \text{ m} = 0.1 \times 1.09 \text{ yd}$$

$$0.1 \text{ m} \approx 0.1 \text{ yd}$$

Find the volume in cubic yards.

$$V = \ell wh$$

$$V = 11.8 \times 10.4 \times 0.1$$

$$V = 12.3 \text{ yd}^3$$

Paulino will need to round up to the nearest 0.5 cubic yard, so he will need  $12.5 \text{ yd}^3$ .

Find the cost by multiplying the number of cubic yards by the cost per cubic yard.

$$12.5 \times \$18.75 = \$234.38$$

It will cost approximately \$234.38 for the topsoil. This answer is different from the first answer because you round at a different point in the calculation.

**BUILD YOUR SKILLS**

4. A garden bed is 4' by 3', and a 6" layer of soil will be spread over the garden. A bag of soil contains  $2 \text{ ft}^3$  of soil. How many bags are needed to cover the garden?
5. Karl buys bales of hay that measure  $15'' \times 24'' \times 36''$ . He needs to buy 250 bales, and he needs to know if they will fit in his barn. What is the total volume of hay in cubic feet?
6. A computer measures 9 inches by 16 inches by  $16\frac{1}{2}$  inches. It requires  $1\frac{1}{2}$  inches of padding on each side to protect it during shipping. What is the volume of the packed box?

## NEW SKILLS: WORKING WITH CAPACITY

The **capacity** of a container is the amount it can hold. Capacity is the volume of a container. Capacity is often used with liquid measures.

In the SI, the basic unit of capacity is the litre. A litre is one one-thousandth of a cubic metre, or 1000 cubic centimetres.

In imperial units, capacity is measured in gallons.

$$4 \text{ quarts} = 1 \text{ gallon}$$

$$2 \text{ pints} = 1 \text{ quart}$$

$$2 \text{ cups} = 1 \text{ pint}$$

However, the gallon has two different sizes:

- The British gallon is approximately 4.5 litres and 1 pint is 20 fluid ounces.
- The American (US) gallon is approximately 3.8 litres and 1 pint is 16 fluid ounces.

In measuring liquids for recipes, the US system is often used.

$$1 \text{ teaspoon (tsp)} = 5 \text{ millilitres (mL)}$$

$$1 \text{ tablespoon (tbsp)} = 15 \text{ mL}$$

$$1 \text{ cup} = 250 \text{ mL}$$

For more information, see page 124 of *MathWorks 10*.

**capacity:** the maximum amount that a container can hold

1 cup is actually 237 mL, but in cooking it is rounded to 250 mL for easier measuring.

**Example 3**

Paula is opening a French bakery and wants to make authentic French recipes. All the recipes are given in metric units, but she has imperial measuring devices. The crème brûlée recipe requires 500 mL of cream and 1.25 mL of vanilla.

- How much cream will she need, in cups?
- How much vanilla will she need, in teaspoons?
- How much cream will she need, in fluid ounces?

**SOLUTION**

- a) Convert 500 mL to cups.

$$1 \text{ cup} = 250 \text{ mL}$$

$$500 \text{ mL} = 500 \text{ mL} \times \frac{1 \text{ cup}}{250 \text{ mL}}$$

$$500 \text{ mL} = 2 \text{ cups}$$

She will need 2 cups of cream.

- b) Convert 1.25 mL to teaspoons.

$$1 \text{ tsp} = 5 \text{ mL}$$

$$1.25 \text{ mL} = 1.25 \text{ mL} \times \frac{1 \text{ tsp}}{5 \text{ mL}}$$

$$1.25 \text{ mL} = 0.25 \text{ tsp or } \frac{1}{4} \text{ tsp}$$

She will need  $\frac{1}{4}$  tsp of vanilla.

- c) Convert 500 mL to fluid ounces.

$$500 \text{ mL} = 2 \text{ cups}$$

$$2 \text{ cups} = 1 \text{ pint}$$

$$1 \text{ pint} = 16 \text{ fl oz}$$

She will need 16 fl oz of cream.

**BUILD YOUR SKILLS**

7. Serina is travelling through the US and her car's gas tank has a capacity of 55 litres.
- How much is this in American gallons?
  - If gas costs \$2.99/gal in Bellingham, WA, how much will it cost to fill her car (assuming that it is totally empty)?
  - Assuming she has the same car in London, England, where gas costs \$9.86/gal (converted from pounds), how much will it cost to fill her tank? (Remember that the British gallon is a different size.)
8. The box of Jakob's cube van has inside dimensions of 20 feet (length), 10 feet 8 inches (width), and 12 feet 6 inches (height). Calculate the volume of the interior.



9. Bev has two storage bins for grain. The first bin is 12 feet 8 inches by 8 feet 9 inches and is filled to a level height of 4 feet 6 inches. If she has to move the grain to a bin with a base measuring 9 feet by 9 feet, what will be the level height of the grain in the second bin?

**PRACTISE YOUR NEW SKILLS**

1. A storage container measures 6 feet by 3 feet by 4 feet.
  - a) What is the volume in cubic feet?
  
  
  
  
  
  
  
  
  
  
  - b) What is the volume in cubic yards?
  
  
  
  
  
  
  
  
  
  
2. A recipe calls for  $2\frac{3}{4}$  cups of milk. How much is this in mL?



6. The exterior of a concrete container will be 10 feet by 8 feet by 4 feet tall. The walls and the bottom are 6 inches thick. What will it cost to construct it if concrete is \$98.95/cubic yard?

**CHAPTER TEST**

#1-9

1. The tallest person in the world was Robert Pershing Wadlow. At the time of his death in 1940, he was 8'11.1" tall. The record for the world's shortest adult was held by He Pingping; at the time of his death in 2010, he was 2'4.7".

a) What is the difference between their heights in feet and inches?

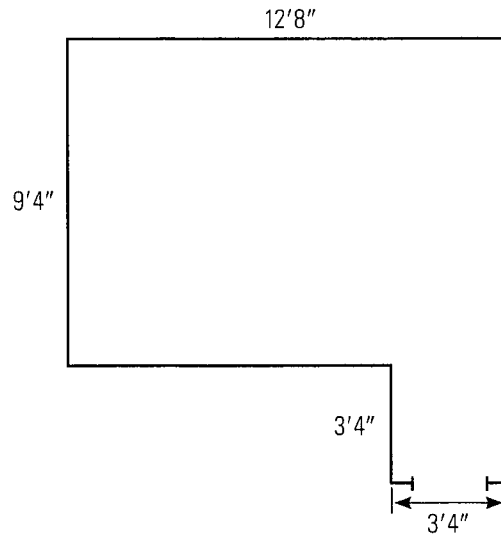
b) Find the height of each man in metres.

c) What is the difference between their heights in metres?

2. While driving in the United States, Franklin sees that the height of a tunnel is marked as  $10'6''$ . He knows that his truck is  $3.3$  m tall. Can he drive through the tunnel?

3. Rachelle is buying panelling for wainscoting for her hall. The panels are  $4'$  by  $8'$ . The wainscoting will be  $4'$  high and the room is  $19'$  by  $13'$ . There are two doors measuring  $30''$  wide, and one  $12'$  window that will only need a  $2'$  panel of wainscoting under it. How many panels will she need, assuming no wastage?

4. Mario is laying tiles for the patio below and planting daffodils around the perimeter.



- a) Assuming he needs to buy 10% more than the area due to wastage, how many  $12''$  by  $12''$  tiles will he need?
- b) How many daffodils will he plant around the perimeter if there are no daffodils along the entrance and he plants them approximately 1 foot apart?

5. Louise needs to give the exterior of a cylindrical granary 2 coats of paint. If the granary is 10 feet tall and has a diameter of 14 feet, and paint covers approximately 375 square feet per gallon, how many gallons of paint will she need to buy? Assume that she can only buy full gallons, and will not be painting the roof.

6. Roberto is painting the exterior of a rectangular storage unit to protect it from rusting. If the unit is 7'6" wide, 9'8" long, and 8'2" tall, what is the surface area in square feet? Roberto will be painting the sides and the roof.

7. A soccer field is 109 m long and 73 m wide. American soccer league rules state that a field should be no more than 120 yards long and 80 yards wide. Is the field within the specified dimensions?

8. What is the volume of water in a fish tank that is 90 cm by 55 cm if it is filled to a height of 32 cm?

9. A driveway is 36 ft long and 10 ft wide, and will be covered in gravel that is 2 in deep. How many cubic yards of gravel will be needed?



10. In the US, milk is commonly sold in jugs of 1 gal,  $\frac{1}{2}$  gal, 1 quart, and  $\frac{1}{2}$  pint. What are the equivalent sizes in millilitres?

11. A recipe for pumpkin cheesecake calls for a 5-US fl oz can of evaporated milk.

a) What is this in cups?

b) What is this in mL?

# 10.

## Math Works Workbook

### Part 2

